Decoherence of the Superconducting Persistent Current Qubit

Lin Tian[1], L.S. Levitov[1], Caspar H. van der Wal[4], J.E. Mooij[2,4], T.P. Orlando[2], S. Lloyd[3], C. J. P. M. Harmans[4], J.J. Mazo[2,5]

[1] *Dept. of Physics, Center for Material Science & Engineering,* [2] *Dept. of Electrical Engineering and Computer Science,* [3] *Dept. of Mechanical Engineering, Massachusetts Institute of Technology;* [4] *Dept. of Applied Physics and Delft Institute for Microelectronics and Submicron Technologies, Delft Univ. of Technology;* [5] *Dept. de Física de la Mataeria Condensada, Universidad de Zaragoza*

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Decoherence of a solid state based qubit can be caused by coupling to microscopic degrees of freedom in the solid. We lay out a simple theory and use it to estimate decoherence for a recently proposed superconducting persistent current design. All considered sources of decoherence are found to be quite weak, leading to a high quality factor for this qubit.

I. INTRODUCTION

The power of quantum logic [1] depends on the degree of coherence of the qubit dynamics [2,3]. The so-called "quality factor" of the qubit, the number of quantum operations performed during the qubit coherence time, should be at least 10^4 for the quantum computer to allow for quantum error correction [4]. Decoherence is an especially vital issue in solid state qubit designs, due to many kinds of low energy excitations in the solid state environment that may couple to qubit states and cause dephasing.

In this article we discuss and estimate **some of the main sources of** decoherence in the superconducting persistent current qubit proposed recently [3]. The approach will be presented in a way making it easy to generalize it to other systems. We emphasize those decoherence mechanisms that illustrate this approach, and briefly summarize the results of other mechanisms.

The circuit [3] consists of three small Josephson junctions which are connected in series, forming a loop, as shown in Fig. 1. The charging energy of the qubits $E_C = e^2/2C_{1,2}$ is ~ 100 times smaller than the Josephson energy $E_J = \hbar I_0/2e$, where I_0 is the qubit Josephson critical current. The junctions discussed in [3] are $200\,\mathrm{nm}$ by $400\,\mathrm{nm}$, and $E_J \approx 200\,\mathrm{GHz}$.

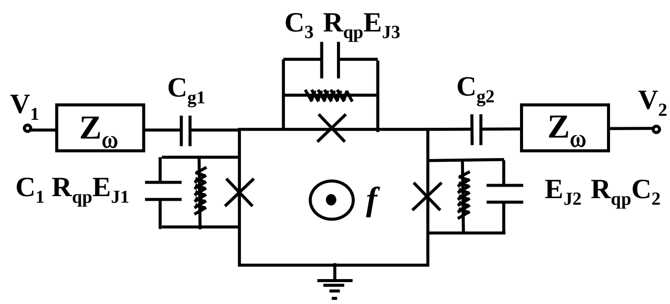

FIG. 1. Schematic qubit design [3] consisting of three Josephson junctions connected as shown. Josephson energy of one of the junctions (number 3 in the figure) is adjustable by varying the flux in the SQUID loop. The impedances Z_ω model electroma gnetic environment coupled to the qubit via gate capacitances $C_{g(1,2)}$. Shunt resistors model quasiparticle subgap resistance effect.

Qubit is realized by two lowest energy states of the system corresponding to opposite circulating currents in the loop. The energy splitting of these states $\varepsilon_0 \approx 10\,\mathrm{GHz}$ is controlled by the external magnetic field flux f, the barrier height is $\simeq 35$ GHz and the tunneling amplitude between the two states is $t \approx 1$ GHz. The Hamiltonian derived in [3] for the two lowest energy levels of the qubit has the form

$$\mathcal{H}_0 \;=\; \begin{pmatrix} -\varepsilon_0/2 & t(q_1,q_2) \\ t^*(q_1,q_2) & \varepsilon_0/2 \end{pmatrix} \;, \tag{1}$$

where $t(q_1,q_2)$ is a periodic function of gate charges $q_{1,2}$. In the tight binding approximation [3], $t(q_1,q_2) = t_1 + t_2 e^{-i\pi q_1/e} + t_2 e^{i\pi q_2/e}$, where t_1 is the amplitude of tunneling between the nearest energy minima and t_2 is the tunneling between the next nearest neighbor minima in the model [3]. Both t_1 and t_2 depend on the energy barrier height and width exponentially. With the parameters of our qubit design, $t_2/t_1 < 10^{-3}$, the effect of fluctuations of $q_{1,2}$ should be small.

Below we consider a number of decoherence effects which seem to be most relevant for the design [3], trying to keep the approach general enough, so that it can be applied to other designs.

II. BASIC APPROACH

We start with a Hamiltonian of a qubit coupled to environmental degrees of freedom in the solid: $\mathcal{H}_{\rm total} = \mathcal{H}_{\rm Q}(\vec{\sigma}) + \mathcal{H}_{\rm bath}(\{\xi_\alpha\})$, where $\mathcal{H}_{\rm Q} = \mathcal{H}_0 + \mathcal{H}_{coupling}$:

$$\mathcal{H}_{\rm Q} = \frac{\hbar}{2}\left(\vec{\Delta}(t) + \vec{\eta}(t)\right)\cdot\vec{\sigma}\ , \qquad \vec{\eta} = \sum_\alpha \vec{A_\alpha}\hat{\xi}_\alpha\ , \tag{2}$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli matrices acting on the qubit states, the vector $\vec{\Delta}$ represents external control, and $\vec{\eta}$ is noise due to coupling to the bath variables ξ_α. In (1), $\Delta_z = -\varepsilon_0$, $\Delta_x - i\Delta_y = t(q_1, q_2)$.

The degrees of freedom that may decohere qubit dynamics are:

- charge fluctuations in the gates coupling qubit states to other states;
- quasiparticles in the superconductor giving rise to subgap resistance;
- nuclear spins in the solid creating fluctuating magnetic fields;
- electromagnetic radiation causing damping of Rabi oscillations;
- coupling between qubits affecting operation of an individual qubit.

In all cases except the last one, the qubit is coupled to a macroscopic number of degrees of freedom $N \gg 1$ with about the same strength A_α to each. In such a situation, the qubit decoherence rate is much larger than the characteristic individual coupling frequency $A_\alpha/\hbar$. This means that dephasing happens on a shorter time scale than it would have taken to create an entangled state of the qubit and one particular element of the bath. In other words, on the decoherence time scale each element of the bath remains in its initial state with probability $1 - O(1/N)$, and it is only due to a large number of relevant degrees of freedom N that the state of the qubit is significantly affected on this time scale.

This observation makes the analysis quite simple, especially because the condition $N \gg 1$ allows one to replace generally noncommuting quantum variables $\widehat{\xi_\alpha}(t)$ by bosonic fields $\eta_{x,y,z}(t)$ fluctuating in time. (Because at large N the commutators $[\eta_i(t), \eta_j(t)]$ are well approximated by $c-$numbers.) As a result, the problem becomes equivalent to that of longitudinal and transverse spin relaxation times T_1 and T_2 in NMR, corresponding to the noise $\eta_i(t)$ either flipping the qubit spin, or contributing a random phase to the qubit states evolution, respectively. Thus we can use the standard Debye–Bloch theory of relaxation in two-level systems.

To adapt this theory to our problem, we assume, without loss of generality, that $\vec{\Delta}(t) \parallel \hat{z}$ and is constant as a function of time. Then one can eliminate the term $\frac{1}{2}\vec{\Delta}\cdot\vec{\sigma}$ by going to the frame rotating around the $z-$axis with the Larmor frequency $\Delta = |\vec{\Delta}|$. In the rotating frame the Hamiltonian (2) becomes:

$$\widetilde{\mathcal{H}}_{\rm Q} = \frac{\hbar}{2}\ \left(\eta_{\|}(t)\sigma_z + e^{-i\Delta t}\eta_{\perp}(t)\sigma_+ + e^{i\Delta t}\eta_{\perp}^*(t)\sigma_-\right)\ , \tag{3}$$

where $\eta_{\|}(t)$ and $\eta_{\perp}(t)$ correspond to components of vector $\vec{\eta}(t)$ in (2) parallel and perpendicular to $\vec{\Delta}$, respectively.

The time evolution due to noise $\vec{\eta}(t)$ is given by the evolution operator $T\exp\left(-i\int\widetilde{\mathcal{H}}_{\rm Q}(t')dt'\right)$ written in the rotating Larmor basis. However, for a simple estimate below we ignore noncommutativity of different parts of the Hamiltonian (3), and consider a c-number phase factor instead of an operator exponent.

Then the decoherence can be characterized using the function

$$R(t) = \max\left[\langle\phi_{\|}^2(t)\rangle,\ \langle|\phi_{\perp}(t)|^2\rangle\right]\ , \tag{4}$$

where $\langle ...\rangle$ stands for ensemble average, and

$$\phi_{\|}(t) = \int\limits_0^t \eta_{\|}(t')dt'\ ,\qquad \phi_{\perp}(t) = \int\limits_0^t e^{-i\Delta t'}\eta_{\perp}(t')dt' \tag{5}$$

The function $R(t)$ grows with time, and one can take as a measure of decoherence the time τ for which $R(\tau)\simeq 1$. There are several assumptions implicit in this criterion. First, we ignore noncommutativity of different terms in (3), which is legitimate at short times, when $R(t)\ll 1$. Second, we include in (4) the zero-point fluctuations of $\widehat{\xi}_\alpha(t)$. The issue of decoherence due to zero-point motion in some cases can be subtle. However, since including the zero-point fluctuations in $R(t)$ can only overestimate the rate of loosing coherence, one expects the criterion $R(\tau)\simeq 1$ to still give a good lower bound on decoherence time.

Finally, we note that (4) contains statistical average over an ensemble of bath realizations. Hence care needs to be taken in the interpretation of τ when the bath is "frozen" into a particular configuration so that the ensemble averaging does not apply. In this situation one has to distinguish between decohering individual qubit dynamics and averaged dynamics of a qubit array. An example of such a situation is provided by the problem of coupling to the nuclear spins, a system with long relaxation times.

Since $\vec{\eta} = \sum\limits_\alpha \vec{A}_\alpha\hat{\xi}_\alpha(t)$, it is the time evolution of $\widehat{\xi}_\alpha(t)$ defined by $\mathcal{H}_{\rm bath}$ that is what eventually leads to decoherence. One can express quantities of interest in terms of the noise spectrum of the components of $\vec{\eta}$:

$$\langle\phi_{\|}^2(t)\rangle = \int d\omega\frac{|1-e^{i\omega t}|^2}{2\pi\omega^2}\langle\eta_{\|}(-\omega)\eta_{\|}(\omega)\rangle \tag{6}$$

$$\langle|\phi_{\perp}(t)|^2\rangle = \int d\omega\frac{|1-e^{i\omega t}|^2}{2\pi\omega^2}\langle\eta_{\perp}(-\omega-\Delta)\eta_{\perp}(\omega+\Delta)\rangle \tag{7}$$

In thermal equilibrium, by virtue of the Fluctuation–Dissipation theorem, the noise spectrum in the RHS of (6) and (7) can be expressed in terms of the out-of-phase part of an appropriate susceptibility.

III. ESTIMATES FOR PARTICULAR MECHANISMS

Here we discuss the above listed decoherence mechanisms and use the expressions (6) and (7) to estimate the corresponding decoherence times. We start with the effect of **charge fluctuations on the gates** due to electromagnetic coupling to the environment modeled by an external impedance Z_ω (see Fig. 1), taken below to be of order of 400 Ω, the vacuum impedance.

The dependence of the qubit Hamiltonian on the gate charges $q_{1,2}$ is given by (1), where $q_{1,2}$ vary in time in response to the fluctuations of gate voltages, $\delta q_{1,2} \approx C_g \delta V_{g(1,2)}$, where the gate capacitance is much smaller than the junction capacitance: $C_g \ll C_{1,2}$. The gate voltage fluctuations are given by the Nyquist formula: $\langle \delta V_g(-\omega)\delta V_g(\omega)\rangle = 2Z_\omega \hbar\omega \coth \hbar\omega/kT$.

In our design, $|t(q_1,q_2)| \ll \varepsilon_0$, and therefore fluctuations of $q_{1,2}$ generate primarily transverse noise $\eta_\perp$ in (3), $\eta_\perp(t) \simeq (2\pi/\hbar e)t_2 C_g \delta V_g(t)$. In this case, according to (7), we are interested in the noise spectrum of δV_g shifted by the Larmor frequency Δ. Our typical $\Delta \simeq 10\,\mathrm{GHz}$ is much larger than the temperature $k_B T/h = 1\,\mathrm{GHz}$ at $T = 50\,\mathrm{mK}$, and thus one has $\omega \simeq \Delta \gg kT/\hbar$ in the Nyquist formula.

The Nyquist spectrum is very broad compared to Larmor frequency and other relevant frequency scales, and thus in (7) we can just use the $\omega = \Delta$ value of the noise power. Evaluating $\int |(1-e^{i\omega t})/\omega|^2 d\omega = 2\pi t$, we obtain

$$R(t) = \langle |\phi_\perp(t)|^2\rangle = \frac{2t}{\hbar}\left(\frac{2\pi}{e} t_2 C_g\right)^2 \Delta Z_{\omega=\Delta} \tag{8}$$

Rewriting this expression as $R(t) = t/\tau$, we estimate the decoherence time as

$$\tau = \Delta^{-1}\frac{\hbar}{2e^2} Z^{-1}_{\omega=\Delta}\left(\frac{e^2}{2\pi C_g t_2}\right)^2 \tag{9}$$

where $\hbar/2e^2 \simeq 4\,\mathrm{k}\Omega$. In the qubit design $e^2/2C_g \simeq 100\,\mathrm{GHz}$, and $t_2 \simeq 1\,\mathrm{MHz}$ when $t_2/t_1 \le 10^{-3}$. With these numbers, one has $\tau = 0.1\,\mathrm{s}$.

The next effect we consider is dephasing due to **quasiparticles on superconducting islands**. At finite temperature, quasiparticles are thermally activated above the superconducting gap Δ_0, and their density is $\sim \exp(-\Delta_0/kT)$. The contribution of quasiparticles to the Josephson junction dynamics can be modeled as a shunt resistor, as shown in Fig. 1. The corresponding *subgap resistance* is inversely proportional to the quasiparticle density, and thus increases exponentially at small temperatures: $R_{\rm qp} \approx R_n \exp \Delta_0/kT$, where R_n is the normal state resistance of the junction. For Josephson current $I_0 = 0.2\ \mu$A, $R_n \approx 1.3\,\mathrm{k\Omega}$. At low temperatures the subgap resistance is quite high, and thus difficult to measure [5]. For estimates below we take $R_{\rm qp} = 10^{11}\ \Omega$ which is much smaller than what follows from the exponential dependence for $T = 50\,$mK.

The main effect of the subgap resistance in the shunt resistor model is generating normal current fluctuations which couple to the phase on the junction. The Hamiltonian describing this effect is

$$\mathcal{H}^{\rm qp}_{\rm coupling} = \sum_i \frac{\hbar}{2e}\varphi_i I_i^{\rm qp}(t)\ , \tag{10}$$

where $i=1,2,3$ labels Josephson junctions. Projecting (10) to the two qubit states, one obtains the Hamiltonian (2) with $\eta_z(t) = I_i^{\rm qp}(t)/e$, $\eta_{x,y} = 0$.

The noise spectrum of the quasiparticle current is given by Nyquist formula:

$$\langle I^{\rm qp}(-\omega) I^{\rm qp}(\omega)\rangle = 2R_{\rm qp}^{-1}\hbar\omega\coth(\hbar\omega/kT) \tag{11}$$

After rotating the basis and transforming the problem to the form (3) we have $\eta_\perp(t) \simeq (t_1/\varepsilon_0)\eta_\parallel(t)$, where $\eta_\parallel(t) \simeq I_i^{\rm qp}(t)/e$ since $t_1 \ll \varepsilon_0$.

The analysis of $\langle|\phi_\perp(t)|^2\rangle$ and $\langle|\phi_\parallel(t)|^2\rangle$ is similar to that described above for charge fluctuations on the gates, and one obtains $R_\perp(t) = 2t(t_1/\varepsilon_0)^2\hbar\Delta/(e^2R_{\rm qp})$, and $R_\parallel(t) = 2t\,kT/(e^2R_{\rm qp})$ which gives

$$\tau = \min\left[\tau_\perp, \tau_\parallel\right] = \min\left[\frac{e^2R_{\rm qp}}{2\hbar\Delta}\left(\frac{\varepsilon_0}{t_1}\right)^2,\ \frac{e^2R_{\rm qp}}{2kT}\right] \tag{12}$$

Taking $R_{qp} = 10^{11}\,\Omega$, $T = 50\,$mK, and $\varepsilon_0/t_1 = 100$, the decoherence times are $\tau_\parallel = 1\,$ms and $\tau_\perp = 10\,$ms.

The decoherence effect of **nuclear spins** on the qubit is due to their magnetic field flux coupling to the qubit inductance. Alternatively, this coupling can be viewed as Zeeman energy of nuclear spins in the magnetic field $\vec{B}(r)$ due to the qubit. The two states of the qubit have opposite currents, and produce magnetic field of opposite sign. The corresponding term in (2) is

$$\mathcal{H}_{\rm coupling} = -\sigma_z\sum_{r=r_i}\mu\vec{B}(r)\cdot\vec{\hat{s}}(r) \tag{13}$$

where r_i are positions of nuclei, μ is nuclear magnetic moment and $\widehat{s}(r_i)$ are spin operators.

Nuclei are in thermal equilibrium, and their spin fluctuations can be related to the longitudinal relaxation time T_1 by the Fluctuation-Dissipation theorem. Assuming that different spins are uncorrelated, one has

$$\langle s_\omega(r)s_{-\omega}(r)\rangle = 2k_BT\frac{\chi''(\omega)}{\omega} = \frac{2k_BT_1\chi_0}{1+\omega^2T_1^2}\ , \tag{14}$$

where $\chi_0 = 1/k_B T$ is static spin susceptibility.

The spectrum (14) has a very narrow width set by the long relaxation time T_1. This width is much less then $k_B T$ and Δ. As a result, only longitudinal fluctuations $\eta_\parallel$ survive in (6) and (7). One has

$$\langle \phi_\parallel^2(t) \rangle = \int d\omega \frac{|1 - e^{i\omega t}|^2}{2\pi \hbar^2 \omega^2} \sum_{r=r_i} \mu^2 B^2(r) \langle s_\omega(r) s_{-\omega}(r) \rangle \ . \tag{15}$$

Plugging the spectrum (14) in (15) and integrating, one obtains

$$\begin{aligned} R(t) &= \frac{T_1}{\tau_0^2} \left(|t| - T_1 + T_1 e^{-|t|/T_1} \right), \\ \tau_0 &= \left(\int \frac{2\mu^2}{\hbar^2} n(r) B^2(r) d^3 r \right)^{-1/2}, \end{aligned} \tag{16}$$

where $n(r)$ is the nuclei concentration. The **ensembled-averaged** decoherence time that defined by $R(\tau) \simeq 1$ is then estimated as:

$$\tau = \begin{cases} \tau_0 & \text{for} \quad T_1 > \tau_0 \\ \tau_0^2 / T_1 & \text{for} \quad T_1 < \tau_0 \end{cases} \tag{17}$$

In superconducting Al, nuclear spin relaxation time is strongly varying with temperature: $T_1 \simeq (300/T\,[\mathrm{K}]) e^{\Delta/k_B T}\,\mathrm{s}$. At $T = 50\,\mathrm{mK}$, the time T_1 is of order of minutes, which exceeds all time scales relevant for qubit operation. To estimate τ_0, we use the magneton $\mu \simeq e\hbar/Mc$, where M is proton mass, and $\int B^2(r) d^3 r \simeq 10^{-5} \Phi_0^2 / w$, where $w \simeq 0.5\,\mu\mathrm{m}$ is the thickness of Al wires in the circuit, and $\Phi_0 = hc/2e$ is the flux quantum. The resulting $\tau_0 \simeq 3 \times 10^{-8}\,\mathrm{s} \ll T_1$.

According to (17), one apparently obtains a worryingly short time $\tau = \tau_0$. However, we note that this result corresponds to ensemble averaging, and one should be careful in applying it to an individual qubit.

The physical picture is that the nuclei spin configuration stays the same over times $\leq T_1$. At such times the perturbation $\vec{\eta}$ due to spins has essentially no time dependence, and so nuclei can be viewed as sources of random *static* magnetic field. The fluxes of this field induced on the qubits depend on initial conditions, and are uncorrelated for different qubits. Typical value of this flux corresponds to the change in Larmor frequency of order of $\delta\Delta \simeq \tau_0^{-1} \simeq 30\,\mathrm{MHz}$.

To summarize, for an individual qubit the effect of nuclear spins is equivalent to a random detuning caused by random change in Δ. For an ensemble of qubits, there will be a distribution of Larmor frequencies of width $\delta\Delta \simeq 30\,\mathrm{MHz}$, even if all qubits are identical. However, since the qubit phase can be kept coherent within a time $\leq T_1$, an indirect observation of Rabi oscillations is still possible by using the so-called "spin-echo technique."

A similar theory can be employed to estimate the effect due to magnetic impurities. The main difference is that for impurity spins the relaxation time T_1 is typically much shorter than for nuclear spins. If T_1 becomes comparable to the qubit operation time, the ensemble averaged quantities will describe a *real* dephasing of an individual qubit, rather than effects of inhomogeneous broadening, like for nuclear spins.

IV. OTHER MECHANISMS

Some sources of decoherence are not amenable to the basic approach considered above, such as radiation losses which we estimate to have $\tau \simeq 10^3\,\mathrm{s}$.

Another such source of decoherence is caused by the magnetic dipole interaction between the qubits. This **interaction between qubits** is described by

$$\mathcal{H}_{\mathrm{coupling}} = \sum_{i,j} \hbar\lambda_{ij}\sigma_z^{(i)} \otimes \sigma_z^{(j)} \ , \qquad \hbar\lambda_{ij} \approx \frac{\mu_i \mu_j}{|r_i - r_j|^3} \tag{18}$$

This interaction is strongest for nearest neighbors. For a square lattice of qubits with the spacing $R = 10\,\mu\mathrm{m}$, one has the nearest neighbor coupling $\lambda \simeq 6\,\mathrm{kHz}$. The corresponding decoherence time $\tau = \lambda^{-1} \simeq 0.2\,\mathrm{ms}$ is relatively short.

Several alterations of the design can be implemented to reduce the effect of qubit-qubit interaction. One can arrange qubits in pairs with opposite sign of circulating currents. This will eliminate dipole moment of a pair, and reduce coupling between different pairs to a somewhat weaker quadrupole interaction. The same result can be achieved by using a superconducting base plane, in which magnetic dipoles will be imaged by dipoles of opposite sign, which will partially cancel the qubit-qubit coupling. Also, one can detune Larmor frequencies of neighboring qubits, moving them apart by more than λ, which will make couplings (18) off-resonant and reduce their effect.

Unwanted coupling between qubits is a common problem in quantum computers. Sophisticated decoupling techniques that have been developed for NMR designs [6], could equally be used here. The idea is to apply a sequence of single bit operations that effectively average out the coupling Hamiltonian over time. Such methods would also be effective for reducing the coupling to the environment [7]. These techniques are fully compatible with quantum computation and could be used to lengthen significantly the effective coherence times.

V. SUMMARY

Our analysis shows that for the qubit design [3] the decoherence time is limited by qubit-qubit coupling. By using methods discussed above the decoherence time can be made at least 1 ms which for $f_{\mathrm{Rabi}} = 100\,\mathrm{MHz}$ gives a quality factor of 10^5, passing the criterion for quantum error correction.

In addition to the effects we discussed, some other decoherence sources are worth attention, such as low frequency charge fluctuations resulting from electron hopping on impurities in the semiconductor and charge configuration switching near the gates [8]. These effects cause $1/f$ noise in electron transport, and may contribute to decoherence at low frequencies. Also, we left out the effect of the *ac* field coupling the two low energy states of the qubit to higher energy states. Results of our numerical simulations of the coupling matrix in the qubit [3] show that Rabi oscillations can be observed even in the presence of the *ac* excitation mixing the states (to be published elsewhere).

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