Problem set for the 5th week of the course Quantum Physics 1

For the tutorial sessions of 1 and 3 October 2014

Homework, to be made before the werkcollege:
From the book (Griffiths 2nd Ed.) Chapter 4 - 4.1, 4.3, 4.7.

Problems to work on during werkcollege:
Problem W5.1 (this hand out), and from the book Chapter 4 - 4.2, 4.13, 4.16, 4.45. This is the minimal set you need to do.
Other good problems that we selected (we advise you to make these for the topics where you need or like to do extra training): from the book Chapter 4 - 4.14 (has overlap with W5.1), 4.15, 4.17.

For solving some of the problems of this week, you may choose to use the standard integrals listed below here (see also the *Mathematical Formulas* on very final page of the Griffiths book [or as pdf on Nestor]).

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi} \qquad \int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx = \tfrac{1}{2}\sqrt{\pi} \qquad \int_{0}^{\infty} x^3 e^{-x}\,dx = 6$$

$$\int_0^{\pi} \sin^3 x\, \cos^2 x\, dx = \frac{4}{15} \qquad \int_0^{\pi} \sin^5 x\, dx = \frac{16}{15}$$

$$\int \sin^n ax \cos^m ax \, \mathrm{d}x = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(n+m)} + \frac{n-1}{n+m} \int \sin^{n-2} ax \cos^m ax \, \mathrm{d}x \qquad (\text{for } m, n > 0)$$

$$\int \sin ax \cos^n ax \, \mathrm{d}x = -\frac{1}{a(n+1)} \cos^{n+1} ax + C \qquad (\text{for } n \neq -1)$$

$$\int \sin^n ax \, \mathrm{d}x = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax \, \mathrm{d}x \qquad (\text{for } n > 2)$$

$$\int \sin^3 ax \, \mathrm{d}x = \frac{\cos 3ax}{12a} - \frac{3 \cos ax}{4a} + C$$

Problem W5.1

The wave function of the hydrogen atom in the ground state is (we follow the notation from the book)

$$\Psi_{1,0,0} = \sqrt{\frac{1}{\pi r_0^3}} \exp\left(-\frac{r}{r_0}\right) ,$$

where $r_0 = \dfrac{4\pi\varepsilon_0 \hbar^2}{me^2}$ is the Bohr radius.

a) Sketch the probability density associated with $\Psi_{1,0,0}$ as a function of r. Where in space does it have its maximum value?
b) What is the most probable value of r in the ground state?
c) Find the expectation value for the distance between the electron and the nucleus.
d) What is the expectation value (in terms of x-, y- and z-coordinates) for the electron position?