# Why operators?

## First examples of operators used for a classical state.

**With operators you can for example:**

1. **Describe how physical property can be calculated, give a certain state for the system.**
2. **Describe how a state B of a system evolves from a state A.**

**Notation for the state of a particle with mass m, moving in 1 dimension (here a classical state with precise values for X and P):**

$X_1, P_1$

**This blue box means "in de state" with the particle at position $X_1$ and momentum $P_1$.**

**Describe how the final state of a system evolves from an initial state with an operator for time evolution.**

**Define an operator for time evolution of a state**

**(here for the case of a free particle):**

$$\hat{U}(t_{START}, t_{END})$$

**Hat on the U shows that it is an operator.**

**The result of this operator working on a state $\boxed{X_1, P_1}$ is**

$$\hat{U}(t_{START}, t_{END})\ \boxed{X_1, P_1} = \boxed{\left(X_1 + \frac{P_1}{m}\cdot(t_{END} - t_{START})\right), P_1}$$

**Describe how a physical property can be calculated, given a system in a certain state. The example here is for kinetic energy.**

**Define an operator for kinetic energy:**

$$\hat{T}$$

**Hat on the T shows that it is an operator.**

**The result of this operator working an the state** $\boxed{X_1, P_1}$ **is**

$$\hat{T}\;\boxed{X_1, P_1} = \frac{P_1^2}{2m}$$

**However, in quantum mechanics a system can be in multiple classical states at the same time. For example, this state:**

$$c_a \boxed{X_1,P_1} + c_b \boxed{X_2,P_2}$$

**"+" means here "and simultaneously also in this state"**

**$c_a$ gives the weight for being in that state.**

**This state is then also possible!**

$$c_a \boxed{X_1,P_1} + c_b \boxed{X_2,P_2} + c_c \boxed{X_1,P_2}$$

**What is now an easy approach for describe the value(s) for kinetic energy?**

$$\hat{T}\left(c_a \boxed{X_1,P_1} + c_b \boxed{X_2,P_2} + c_c \boxed{X_1,P_2}\right) = c_a \frac{P_1^2}{2m} + c_b \frac{P_2^2}{2m} + c_c \frac{P_2^2}{2m}$$

**N.B., the pure state $\boxed{X_1,P_1}$ is quantum mechanically not allowed by the Heisenberg uncertainty relation. This is neglected for these examples on operators for classical states.**