Problem set for the 5th week of the course Quantum Physics 1

(version 2015-2016, still valid in later years if handed out as a print or provided on the course website)

Homework, to be made before the werkcollege:
From the book (Griffiths 2nd Ed.) Chapter 4 - 4.1, 4.3, 4.7.

Problems to work on during werkcollege:
Problem W5.1 (this hand out), and from the book Chapter 4 - 4.2, 4.13, 4.16, 4.45. This is the minimal set you need to do.
Other good problems that we selected (we advise you to make these for the topics where you need or like to do extra training): from the book Chapter 4 - 4.14 (has overlap with W5.1), 4.15, 4.17.

Note: for this week the problem set is less work than for weeks 6, 7, and 8 of the course. So, this is a good week to rehearse or catch up on questions you still have on weeks 1 – 4, or to already practice some final exam problems (see course web site).

For solving some of the problems of this week, you may choose to use the standard integrals listed below here (see also the *Mathematical Formulas* on the very final page in some versions of the Griffiths book [available as pdf on Nestor]).

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi} \qquad \int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx = \tfrac{1}{2}\sqrt{\pi} \qquad \int_{0}^{\infty} x^3 e^{-x}\,dx = 6$$

$$\int_0^{\pi} \sin^3 x \,\cos^2 x \; dx = \frac{4}{15} \qquad \int_0^{\pi} \sin^5 x \; dx = \frac{16}{15}$$

$$\int \sin^n ax \cos^m ax \,\mathrm{d}x = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(n+m)} + \frac{n-1}{n+m}\int \sin^{n-2} ax \cos^m ax \,\mathrm{d}x \qquad (\text{for } m,n>0)$$

$$\int \sin ax \cos^n ax \,\mathrm{d}x = -\frac{1}{a(n+1)}\cos^{n+1} ax + C \qquad (\text{for } n \neq -1)$$

$$\int \sin^n ax \,\mathrm{d}x = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n}\int \sin^{n-2} ax \,\mathrm{d}x \qquad (\text{for } n>2)$$

$$\int \sin^3 ax \,\mathrm{d}x = \frac{\cos 3ax}{12a} - \frac{3\cos ax}{4a} + C$$

Problem W5.1

The wave function of the hydrogen atom in the ground state is (also presented in the book, Eq. [4.80])

$$\Psi_{1,0,0} = \sqrt{\frac{1}{\pi r_0^3}}\exp\left(-\frac{r}{r_0}\right) \;, \qquad \text{where } r_0 = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} \text{ is the Bohr radius.}$$

a) Sketch the probability density associated with $\Psi_{1,0,0}$ as a function of *r*. Where in space does it have its maximum value? *Hint:* simply plot here the actual value of the probability density that is valid at points in space that you visit when you go away from the nucleus along a straight axis (can be any axis), while calling the distance away from the nucleus *r*.

b) Now take into consideration that the electron wave function is in a three-dimensional space around the nucleus. What is (while accounting for this consideration) the most probable value of *r* in the ground state? *Hints:* Note that $\Psi_{1,0,0}$ is spherically symmetric and only a function of *r*. You need to describe the probability of finding the electron somewhere in the shell between *r* and *r+dr* in the three-dimensional space.

c) Find the expectation value for the distance between the electron and the nucleus.

d) What is the expectation value (in terms of *x*-, *y*- and *z*-coordinates) for the electron position?