Problem set for 1st week of the course Quantum Physics 1

(version 2015-2016, still valid in later years if handed out as a print or provided on the course website)

Homework, to be made before the werkcollege:
From the book (Griffiths 2nd Ed.) Chapter 1 - 1.1, 1.2, 1.3, 1.4, 1.7 and 1.18

Problems to work on during werkcollege:
Problems W1.1 – W1.3 (this hand out), this is the minimal set you need to do.
Other good problems that we selected (we advise you to make these for the topics where you need or like to do extra training): from the book Chapter 1 - 1.5, 1.8, 1.9, 1.11, 1.14, 1.15, 1.17

Useful standard integral

$$\int_{-\infty}^{\infty} e^{-x^2/a^2}\,dx = a\sqrt{\pi}$$

Problem W1.1 (Note: For this problem, assume that the constants A_i are real valued.)

The wavefunctions of particles 1-5 are (in one dimension) given by

$$\Psi_1(x,t) = A_1 e^{-x^2/4}\ ,$$

$$\Psi_2(x,t) = A_2 e^{-x^2/b} e^{3i}\ ,$$

$$\Psi_3(x,t) = A_3 e^{-|2x|}\ ,$$

$$\begin{cases} \Psi_4(x,t) = A_4 e^{i\omega t}, & |x| \le a \\ \Psi_4(x,t) = 0, & |x| > a \end{cases}\ ,$$

$$\begin{cases} \Psi_5(x,t) = A_5 \cos\left(\pi \dfrac{x}{2a}\right) e^{i\omega t}, & |x| \le a \\ \Psi_5(x,t) = 0, & |x| > a \end{cases}\ .$$

a) For all 5 cases, write out the normalized wavefunctions and sketch the probability densities versus x.

b) For Ψ_4, assume a=2, and calculate the probability that measurement of x gives a result between 1 and 2.

c) For Ψ_5, assume a=2, and calculate the probability that measurement of x gives a result between 1 and 2.

d) Give an example of a physical situation that gives a wavefunction in the form of Ψ_5.

Problem W1.2
A superconducting loop can have so little damping that the electrical current in the loop behaves quantum mechanically. We consider here a particular system that can only be in two states: a state in which the current I flows clockwise (+1 μA), or a state in which the current I flows counter clockwise (-1 μA). For both of these states the amplitude of the current is 1 μA. The quantum state of the loop can be a superposition of these two states. The wavefunction for the system in the clockwise state is $\Psi(I,t) = \varphi_R$, for the other state it is $\Psi(I,t) = \varphi_L$.

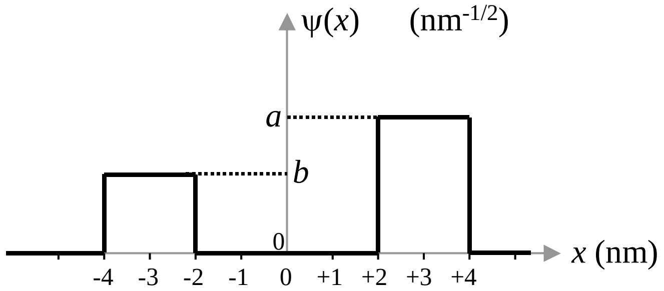
a) The system is prepared in a quantum state such that the probability for it to be in one of the two current states is 50%. Sketch a probability density $P(I) = |\Psi(I,t)|^2$. Write down a normalized wavefunction in terms of φ_R and φ_L that is in agreement with this situation (more answers possible).

b) The apparatus for preparing this state maybe needs to be calibrated. It maybe has an offset (that is stable in time), which causes that the prepared superposition state corresponds to probabilities (50+A)% for the φ_R state and (50-A)% for the φ_L state. To check this, we will prepare the system many times in the same way and measure it. This can be used to see what the value for A is, and one can thus check whether A is very close to zero. The current in the loop is measured by turning on the coupling to a device that can measure the current in the loop. This is done after the preparation step. What are the possible current values on the display of the measuring device?

c) The experimentalists find out that the offset is large, and find the counter-clockwise state 25% of the time. Write down a normalized wavefunction in terms of φ_R and φ_L that is in agreement with this situation (more answers possible).

Problem W1.3
Note: This problem uses the concept *expectation value*, (Eq. [1.28] in the book). You can get the *quantum uncertainty in position* Δx (analogues to a standard deviation) by using Griffiths book Eqs. [1.12] – [1.19], or by using the definition for.given in part b).

The position x of a particle is described by the normalized, real-valued wavefunction as sketched in the figure below, with $a = \sqrt{3/10 \ \text{nm}^{-1}}$ and $b = \sqrt{2/10 \ \text{nm}^{-1}}$.



a) What is the expectation value $\langle \hat{x} \rangle$ for this state?

b) Show that the uncertainty Δx in the particle's position is $\sqrt{\dfrac{5384}{600}}$ nm.

Note: $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$. Many books use the notation Δx, the Griffiths books denotes this as σ_x.

c) With the particle's wavefunction as sketched, you plan to measure the position x. What is the probability for detecting a value in the range $\langle \hat{x} \rangle$ - 0.1 nm < x < $\langle \hat{x} \rangle$ + 0.1 nm?

d) With the particle's wavefunction as sketched, you plan to measure the position x. What is the probability for detecting a value in the range -4 nm < x < -3 nm?

e) Explain the unit of the constant a, and check that you have the units right in questions **a)** – **d)**.