Quantum Physics 1

2015-2016

These slides: The Harmonic Oscillator

Lectures for the 7th and 8th week of the course

Last lectures mainly re-visit Chapter 2 and present
The solid-state physics topics of Chapter 5

Any questions on the material till now?

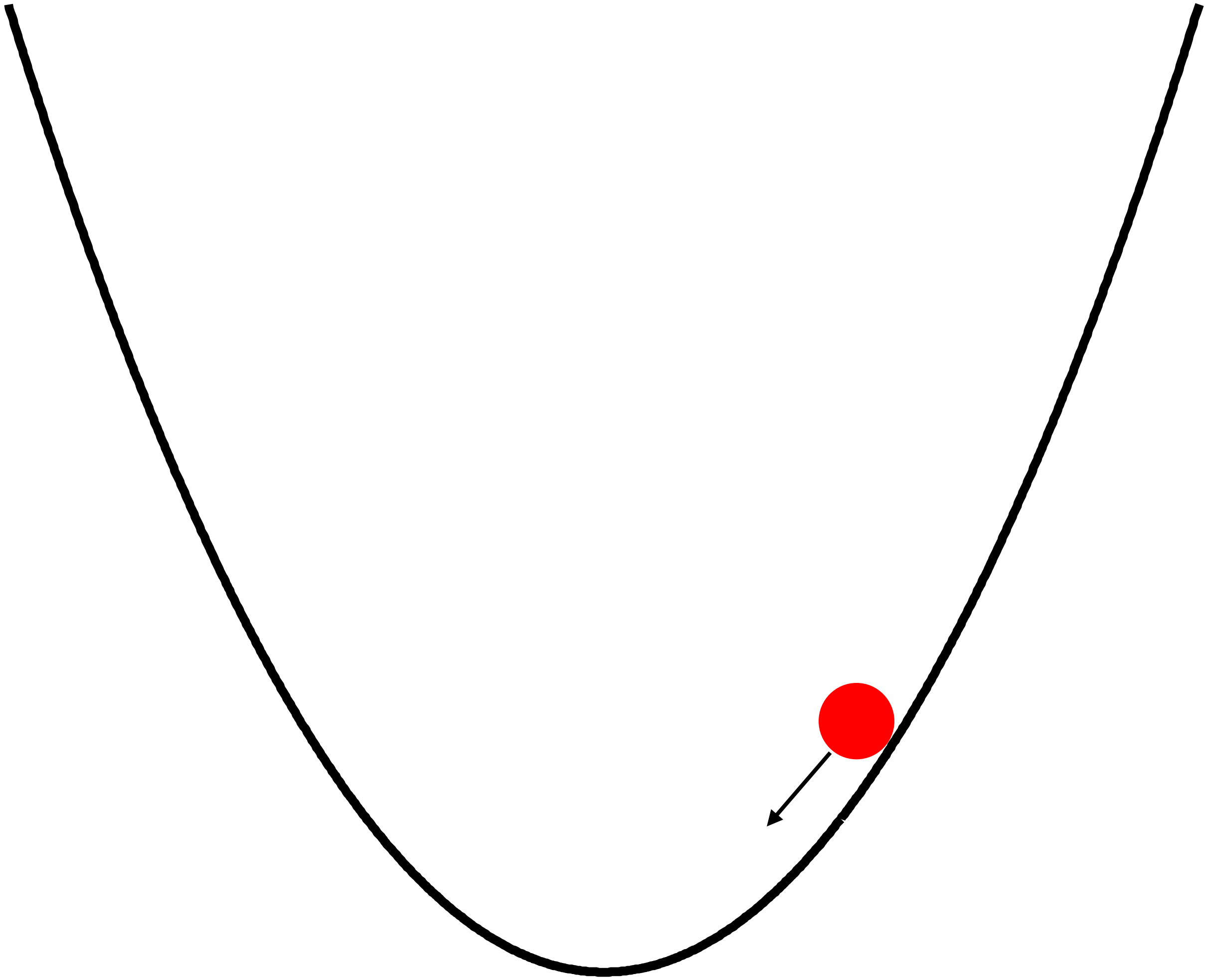
Today

Harmonic oscillators, photons

….vacuum fluctuations

….Casimir effect

1D Harmonic oscillator

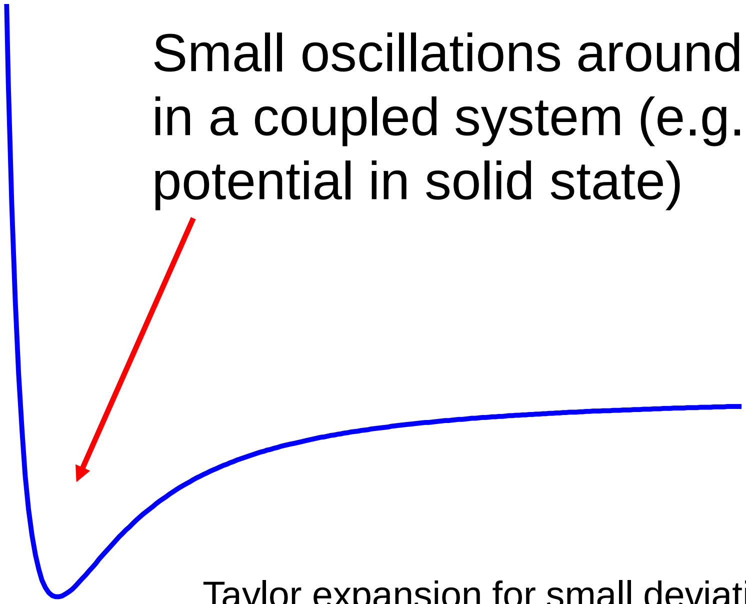


1D Harmonic oscillator
Very important model systems

EM waves (photons)

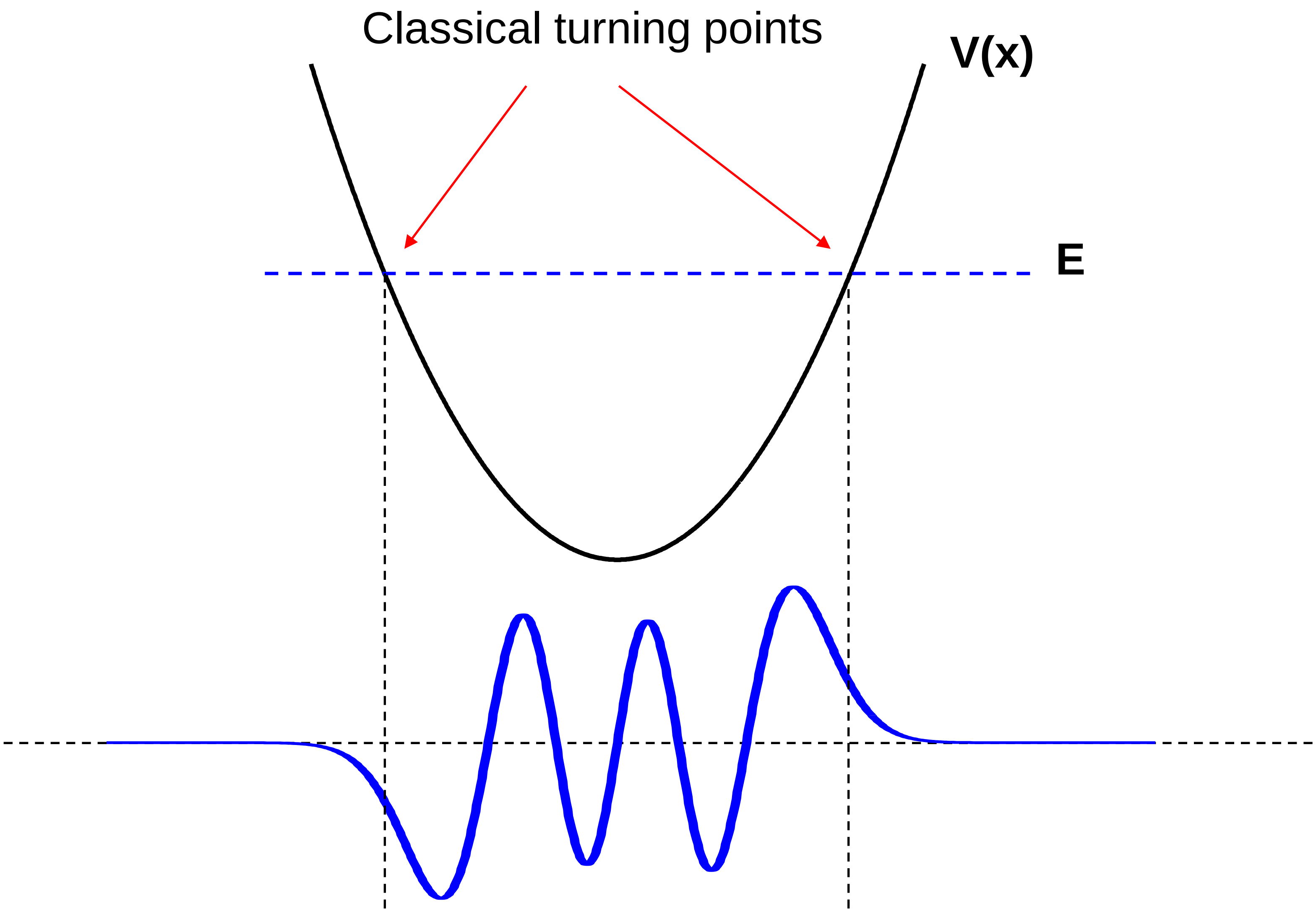
Lattice vibrations (phonons)

Small oscillations around equilibrium in a coupled system (e.g. effective potential in solid state)



$$V(x) = \frac{a}{x^6} - \frac{4a}{x^2}$$

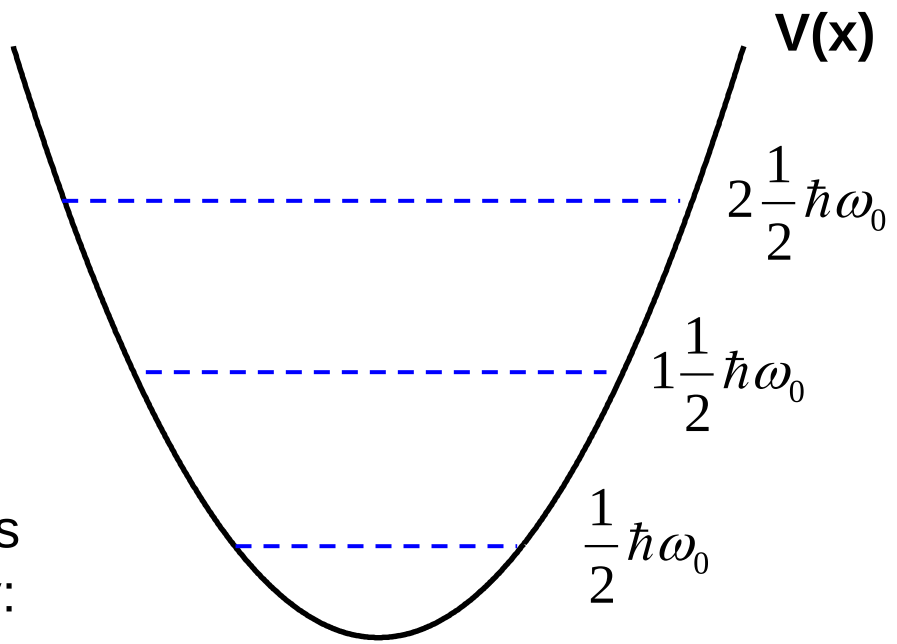
Taylor expansion for small deviations around the x-position of the potential minimum show that the effective potential is here very close to parabolic (Griffiths Eqs. [2.42]-[2.43]).



Local (x-dependent) solutions have the following character:

$$e^{+kx} \qquad e^{+ikx} + e^{-ikx} \qquad e^{-kx}$$

Eigen states and zero-point or vacuum fluctuations



Groundstate has nonzero energy: Zero-point or vacuum fluctuations

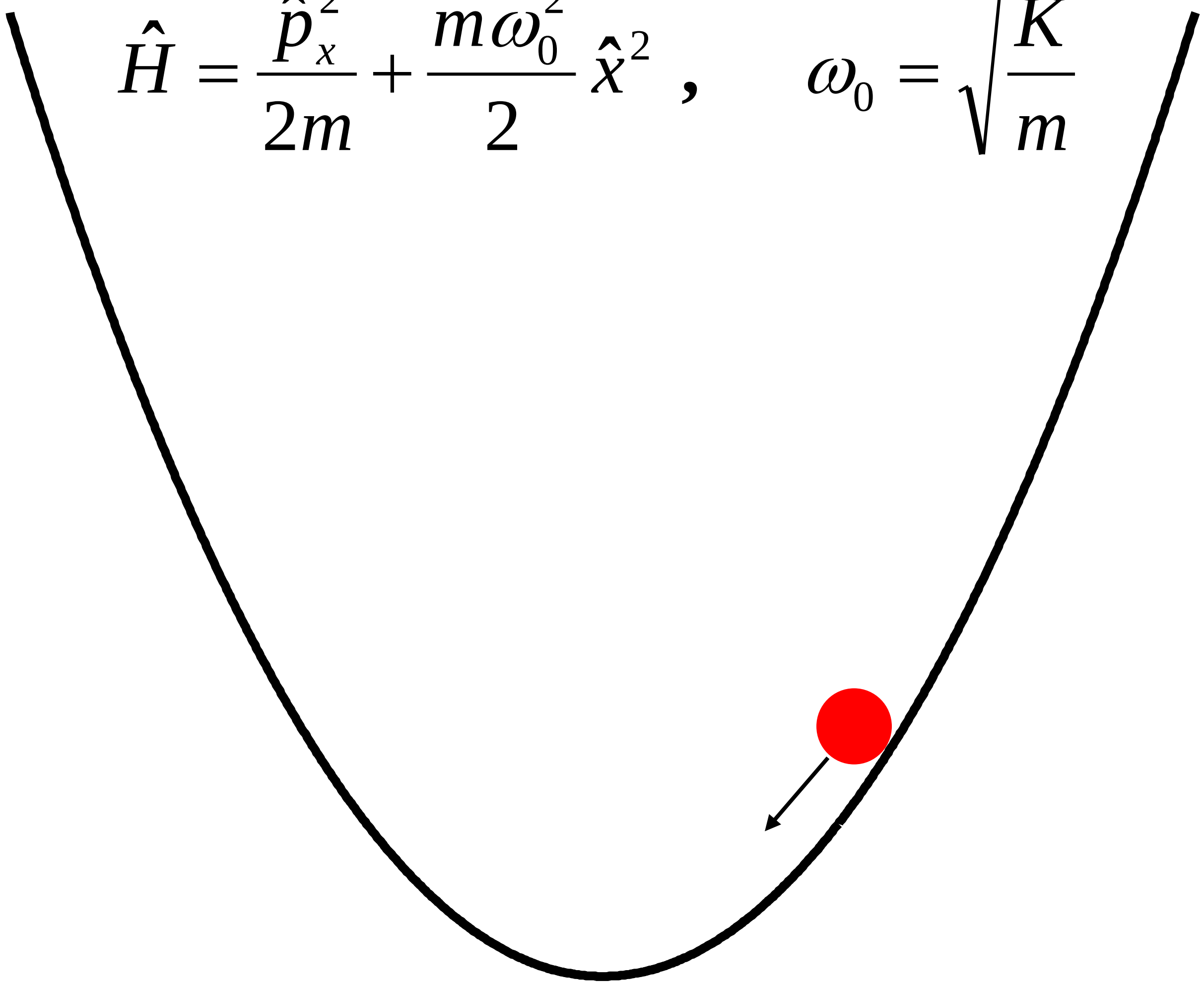
$$\hat{H} = \hbar\omega_0\left(\hat{N} + \frac{1}{2}\right)$$

$$\hat{H}\left|n\right\rangle = \hbar\omega_0\left(n + \frac{1}{2}\right)\left|n\right\rangle$$

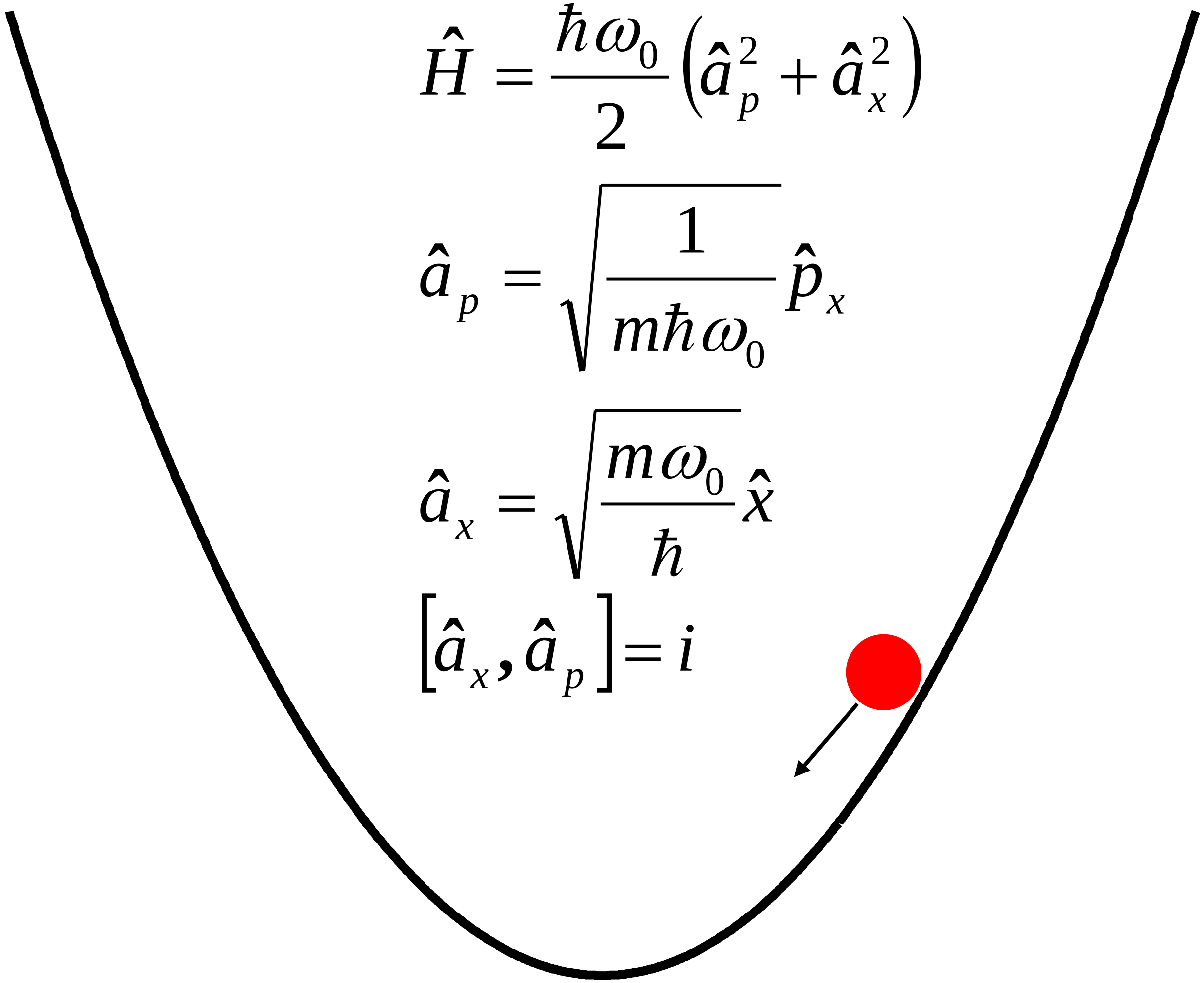
1D Harmonic oscillator

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{K}{2}\hat{x}^2$$

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{m\omega_0^2}{2}\hat{x}^2 \,, \qquad \omega_0 = \sqrt{\frac{K}{m}}$$



1D Harmonic oscillator



$$\hat{H} = \frac{\hbar\omega_0}{2}\left(\hat{a}_p^2 + \hat{a}_x^2\right)$$

$$\hat{a}_p = \sqrt{\frac{1}{m\hbar\omega_0}}\,\hat{p}_x$$

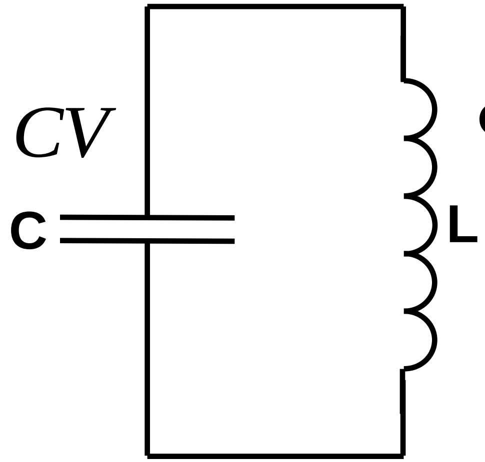
$$\hat{a}_x = \sqrt{\frac{m\omega_0}{\hbar}}\,\hat{x}$$

$$\left[\hat{a}_x, \hat{a}_p\right] = i$$

Any harmonic oscillator can be expressed in a pair of <u>normalized</u>, <u>dimensionless</u>, <u>conjugate</u> coordinates

Other example of harmonic oscillator system that can be mapped on dimensionless coordinates a_x and a_p: LC circuit

$$Q = CV$$



$$\Phi = LI = L\frac{dQ}{dt}$$

$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}, \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

$\dfrac{\hat{Q}^2}{2C}$ kinetic-energy-like term (C is “mass”, Q is “momentum”)

$\dfrac{\hat{\Phi}^2}{2L}$ potential-energy-like term (Φ is “position”)

$$\hat{H} = \frac{\hbar\omega_0}{2}\left(\hat{a}_p^2 + \hat{a}_x^2\right)$$

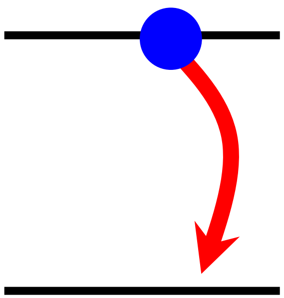
$$\hat{a}_p = \sqrt{\frac{1}{C\hbar\omega_0}}\hat{Q}$$

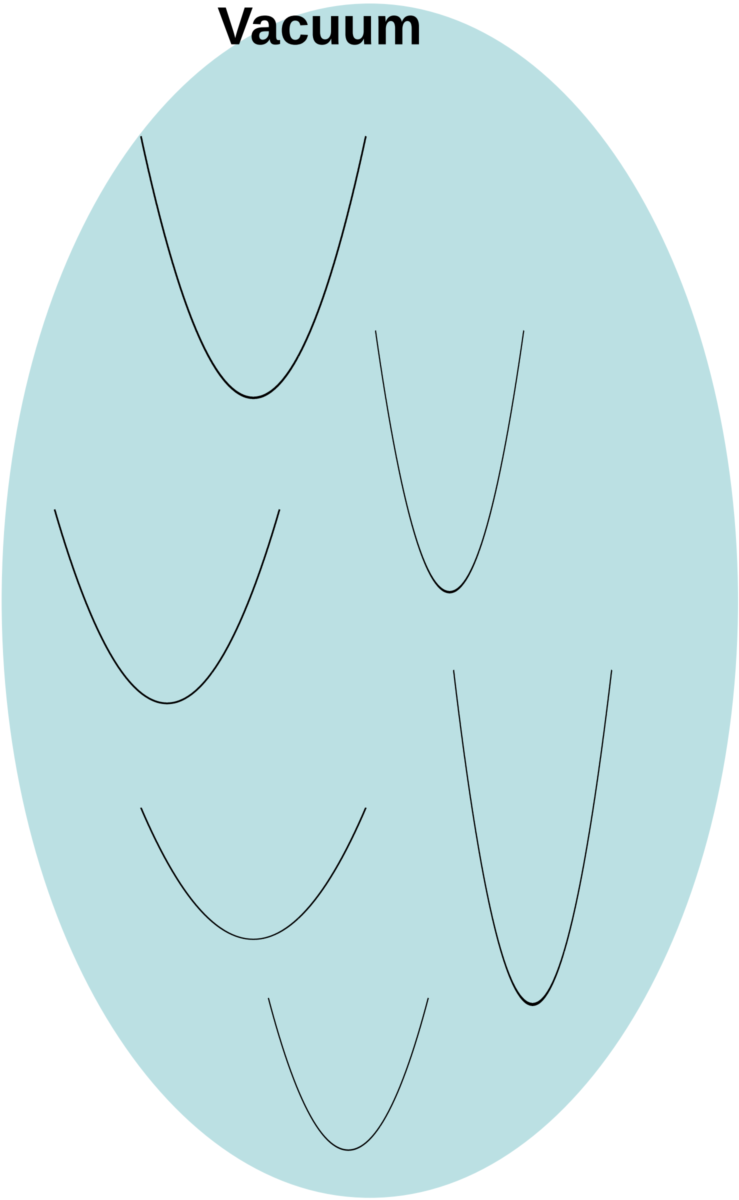
$$\hat{a}_x = \sqrt{\frac{C\omega_0}{\hbar}}\hat{\Phi}$$

$$\left[\hat{a}_x, \hat{a}_p\right] = i$$

Vacuum and spontaneous emission by 2-level systems

Two-level system

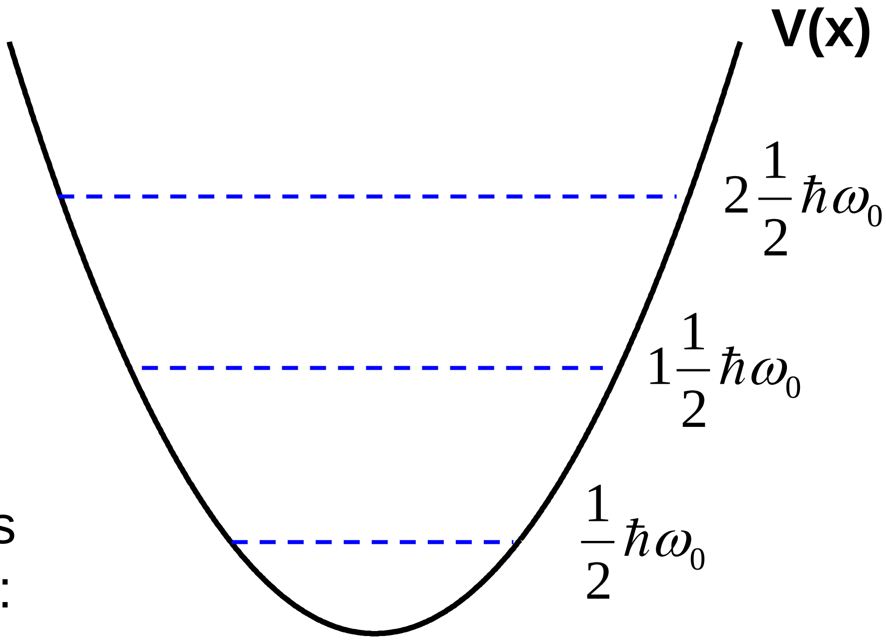




Hamiltonian for oscillating modes of the EM field in vacuum
(here expressed as the energy density of *E* and *B* field)

$$\hat{H} = \frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$$

Eigen states and zero-point or vacuum fluctuations



Groundstate has nonzero energy: Zero-point or vacuum fluctuations

$$\hat{H} = \hbar\omega_0\left(\hat{N} + \frac{1}{2}\right)$$

$$\hat{H}\left|n\right\rangle = \hbar\omega_0\left(n + \frac{1}{2}\right)\left|n\right\rangle$$

$$\begin{cases} \hat{a} = \dfrac{1}{\sqrt{2}}\left(\hat{a}_x + i\hat{a}_p\right) \\ \hat{a}^+ = \dfrac{1}{\sqrt{2}}\left(\hat{a}_x - i\hat{a}_p\right) \end{cases}$$

Annihilation/destruction operator

Creation operator

Non-Hermitian operators!

$$\begin{cases} \hat{a}_x = \dfrac{1}{\sqrt{2}}\left(\hat{a}^+ + \hat{a}\right) \\ \hat{a}_p = \dfrac{i}{\sqrt{2}}\left(\hat{a}^+ - \hat{a}\right) \end{cases}$$

$$\left[\hat{a}, \hat{a}^+\right] = 1$$

Why use this notation?

Algebraic convenience

Physical meaning op creation and annihilation

Note: the notation on these slides differs a bit from the Griffiths book, according to $\hat{a} \rightarrow \hat{a}_-$ and $\hat{a}^+ \rightarrow \hat{a}_+$. I do this on purpose, since the notation used here is much more widely used in the literature.

$$\left[\hat{a},\hat{a}^{+}\right]=1$$

$$\hat{a}\hat{a}^{+}-\hat{a}^{+}\hat{a}=1$$

$$\hat{a}\hat{a}^{+}=\hat{a}^{+}\hat{a}+1$$

$$\hat{a}^{+}\hat{a}=\hat{N}$$

$$\hat{a}\hat{a}^{+}=\hat{N}+1$$

$$\hat{H}=\hbar\omega_0\left(\hat{N}+\frac{1}{2}\right)$$

$$\hat{H}\left|n\right\rangle=\hbar\omega_0\left(n+\frac{1}{2}\right)\left|n\right\rangle$$

Nature of eigensates leads to the concept of PHOTONS

$$\hat{N}\left|n\right\rangle=n\left|n\right\rangle$$

$$\hat{a}\left|n\right\rangle=\sqrt{n}\,\left|n-1\right\rangle$$

Annihilation/destruction operator

- removes a photon from a state

$$\hat{a}^{+}\left|n\right\rangle=\sqrt{n+1}\,\left|n+1\right\rangle$$

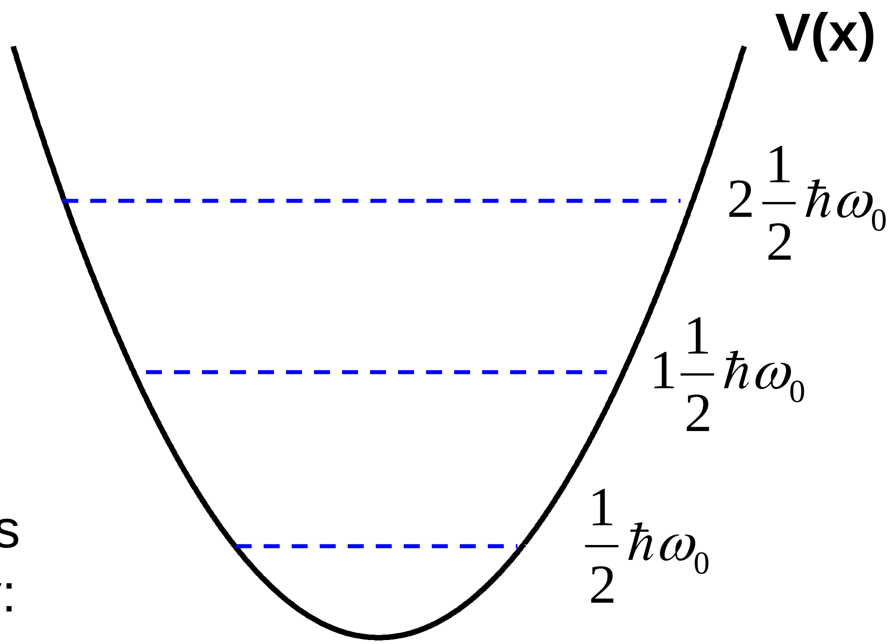
Creation operator

– adds a photon to a state

$$\hat{a}\left|0\right\rangle=0$$

Only positive photon numbers

Eigen states and zero-point or vacuum fluctuations



Groundstate has nonzero energy: Zero-point or vacuum fluctuations

$$\hat{H} = \hbar\omega_0\left(\hat{N} + \frac{1}{2}\right)$$

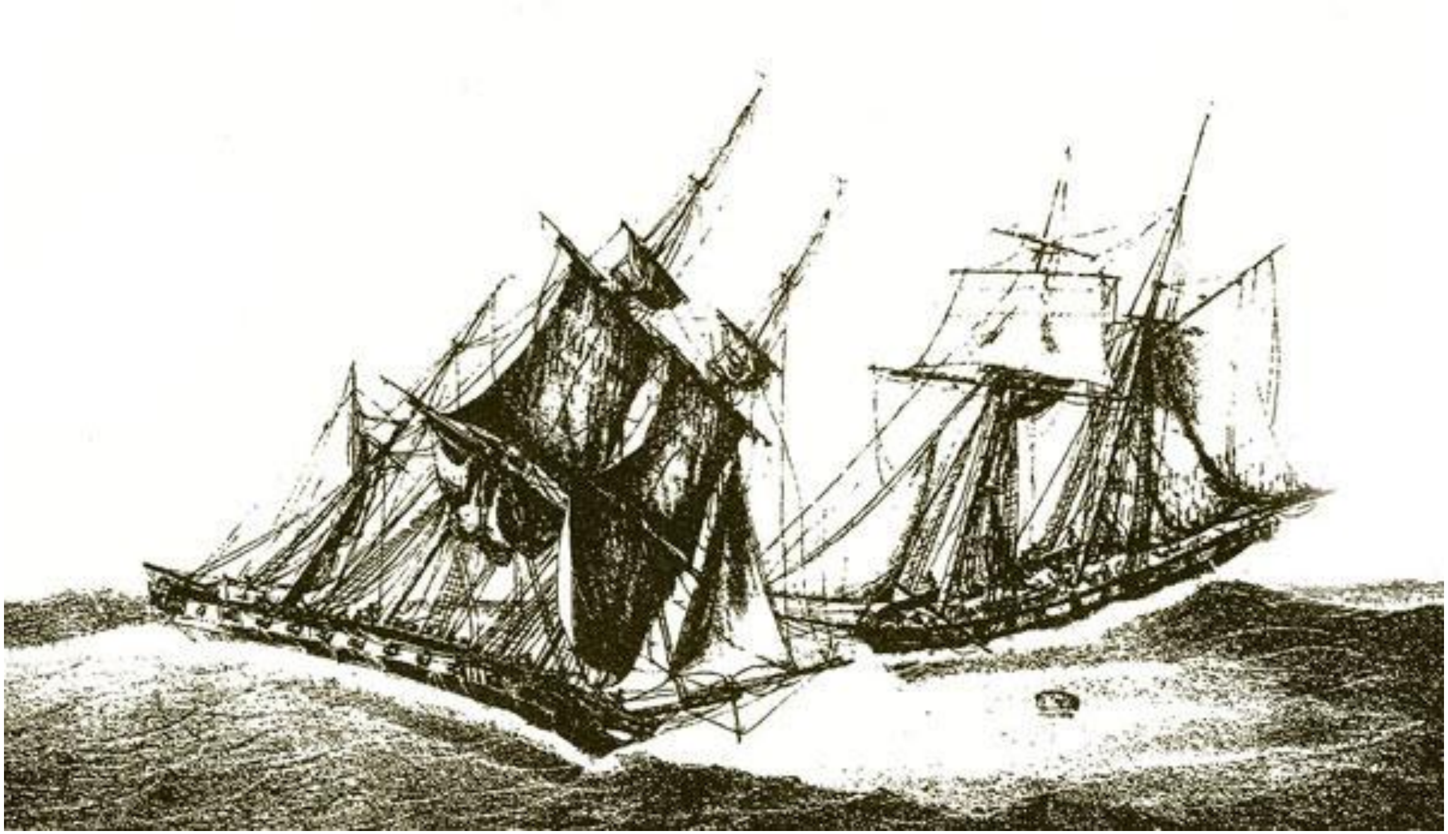
$$\hat{H}|n\rangle = \hbar\omega_0\left(n + \frac{1}{2}\right)|n\rangle$$

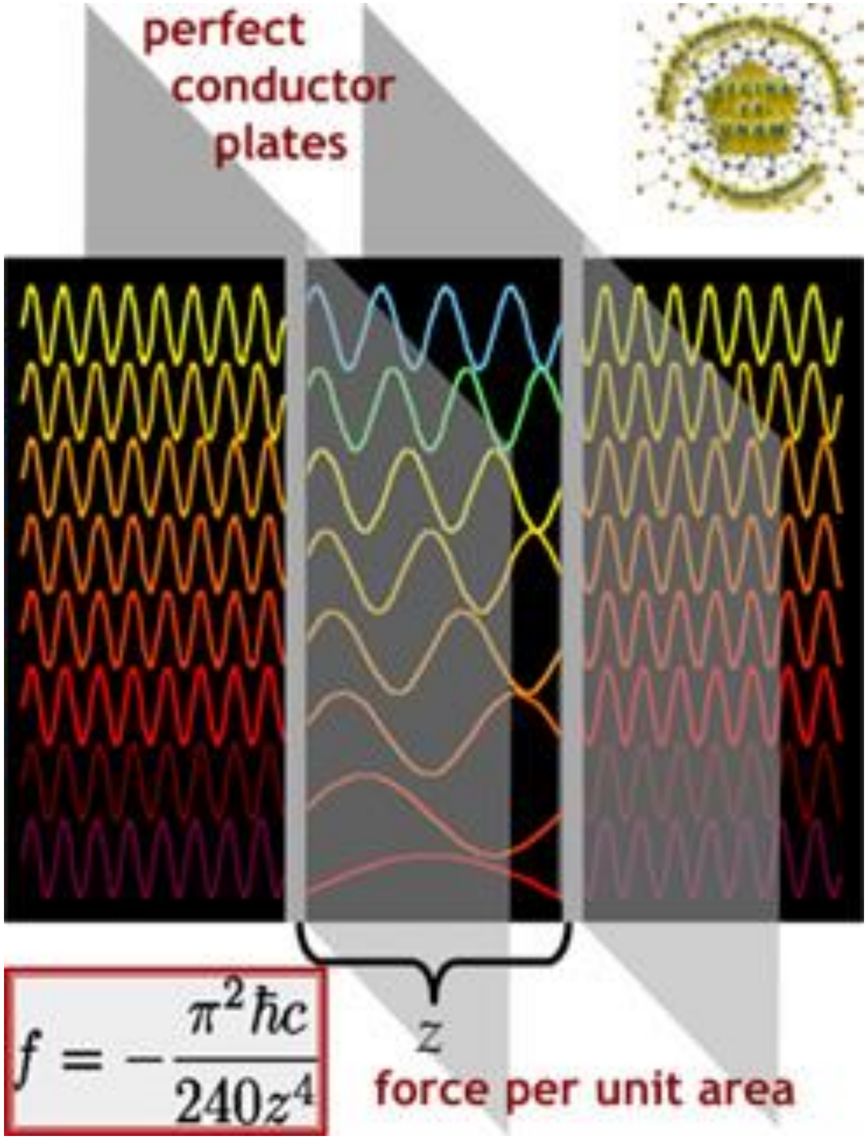
Discussion of Wikipedia page about Jaynes–Cummings model

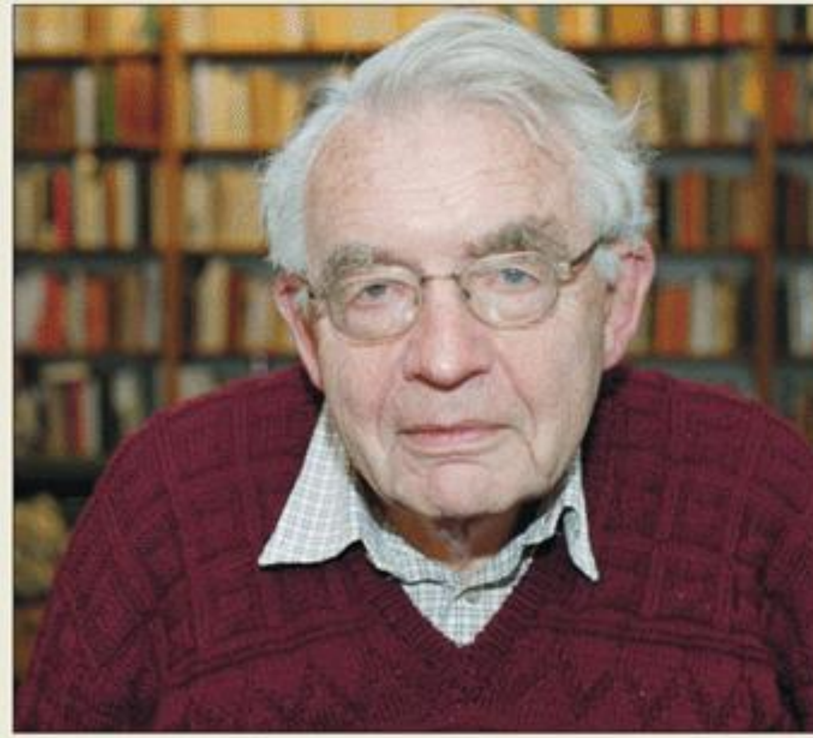
http://en.wikipedia.org/wiki/Jaynes%E2%80%93Cummings_model

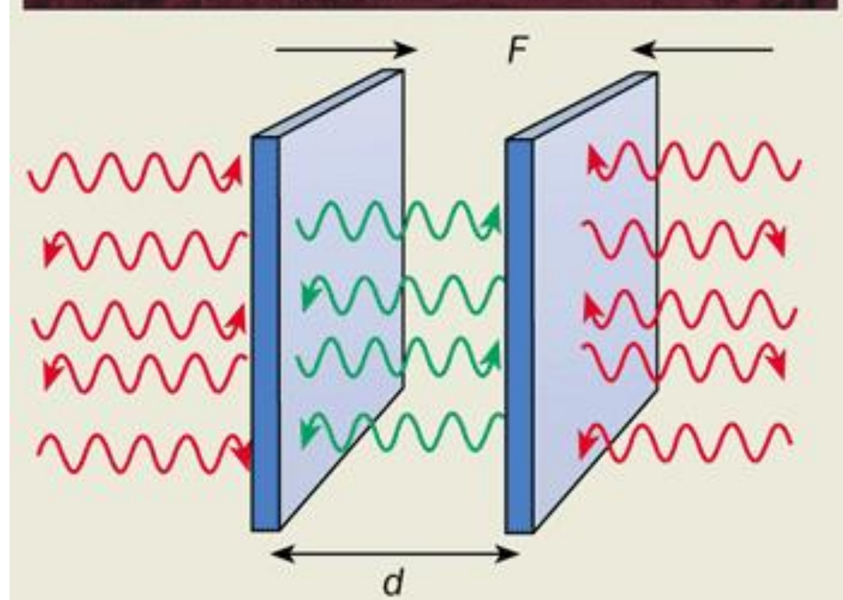
as an example of how raising and lowering operators, and creation and annihilation operators can be very useful. Here it is used when an atom is coupled to a photonic mode.

Casimir effect




perfect
conductor
plates
$f = -\frac{\pi^2 \hbar c}{240 z^4}$
z
force per unit area

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Summary:

Harmonic oscillator, photons

Last 2 lectures:

Wave mechanics, tunneling
Coupling 2 quantum systems: LCAO
From 1D potential well to solid state