Quantum Physics 1

2015-2016

Week 7 – Identical particles

Any questions?

Today

Symmetry and identical particles
System with 2 particles - exchange

But let's start with facts from high school about atoms (Pauli exclusion principle)

… and two double-well potentials far away from each other.

(students calculate P_{LL} for two double-wells that are and always have been far away from each other)

But let's start with facts from high school about atoms:

Pauli exclusion principle

The universe would be very very different without this rule!

Also:

Fits in the big picture of this course.

Your teacher finds this the hardest/weirdest part.

State of total (two-particle) system is a product state of the individual particle states (see Griffiths book Eq. [5.9] (Liboff p. 318))

$$\Psi_T = \Psi_{n_1}\left(x_1\right)\Psi_{n_2}\left(x_2\right)$$

State of total (two-particle) system is a product state of the individual particle states

$$\Psi_T = \Psi_{n_1}(x_1)\Psi_{n_2}(x_2)$$

But we might as well do different labeling 1 and 2 (as it is also an eigen state of the Hamiltoninan with the same eigen energy, and we cannot distinguish the particles).

$$\Psi_T = \Psi_{n_1}(x_2)\Psi_{n_2}(x_1)$$

Only a symmetric or anti-symmetric states with respect to our arbitrary particle labeling does give results where the physics does not depend on our labeling.

$$\Psi_S = \frac{1}{\sqrt{2}}\left[\Psi_{n_1}(x_1)\Psi_{n_2}(x_2) + \Psi_{n_1}(x_2)\Psi_{n_2}(x_1)\right]$$

$$\Psi_A = \frac{1}{\sqrt{2}}\left[\Psi_{n_1}(x_1)\Psi_{n_2}(x_2) - \Psi_{n_1}(x_2)\Psi_{n_2}(x_1)\right]$$

For a system with two identical quantum particles, the state must always be symmetric or anti-symmetric with respect to our arbitrary partcicle labeling (otherwise the state cannot represent real physics)

$$
\begin{aligned}
\hat{H}_T\left|\Psi_T\right\rangle &= \hat{H}_T\left(\alpha\left|\Psi_T\right\rangle_{C1} + \beta\left|\Psi_T\right\rangle_{C2}\right) \\
&= \alpha\,\hat{H}_T\left|\varphi_{g1}\right\rangle\left|\varphi_{e2}\right\rangle + \beta\,\hat{H}_T\left|\varphi_{e1}\right\rangle\left|\varphi_{g2}\right\rangle \\
&= \alpha\left(\hat{H}_1\left|\varphi_{g1}\right\rangle\left|\varphi_{e2}\right\rangle + \hat{H}_2\left|\varphi_{g1}\right\rangle\left|\varphi_{e2}\right\rangle\right) + \beta\left(\hat{H}_1\left|\varphi_{e1}\right\rangle\left|\varphi_{g2}\right\rangle + \hat{H}_2\left|\varphi_{e1}\right\rangle\left|\varphi_{g2}\right\rangle\right) \\
&= \alpha\left(E_g\left|\varphi_{g1}\right\rangle\left|\varphi_{e2}\right\rangle + \left|\varphi_{g1}\right\rangle E_e\left|\varphi_{e2}\right\rangle\right) + \beta\left(E_e\left|\varphi_{e1}\right\rangle\left|\varphi_{g2}\right\rangle + \left|\varphi_{e1}\right\rangle E_g\left|\varphi_{g2}\right\rangle\right) \\
&= \alpha\left(E_g + E_e\right)\left|\varphi_{g1}\right\rangle\left|\varphi_{e2}\right\rangle + \beta\left(E_g + E_e\right)\left|\varphi_{e1}\right\rangle\left|\varphi_{g2}\right\rangle \\
&= \left(E_g + E_e\right)\left(\alpha\left|\Psi_T\right\rangle_{C1} + \beta\left|\Psi_T\right\rangle_{C2}\right).
\end{aligned}
$$

For discussing some of the longer equations in this week's handout

$$\begin{cases} |\varphi_g\rangle = \frac{1}{\sqrt{2}}\left(|\varphi_L\rangle + |\varphi_R\rangle\right) \\ |\varphi_e\rangle = \frac{1}{\sqrt{2}}\left(|\varphi_L\rangle - |\varphi_R\rangle\right) \end{cases} \Leftrightarrow \begin{cases} |\varphi_L\rangle = \frac{1}{\sqrt{2}}\left(|\varphi_g\rangle + |\varphi_e\rangle\right) \\ |\varphi_R\rangle = \frac{1}{\sqrt{2}}\left(|\varphi_g\rangle - |\varphi_e\rangle\right) \end{cases}$$

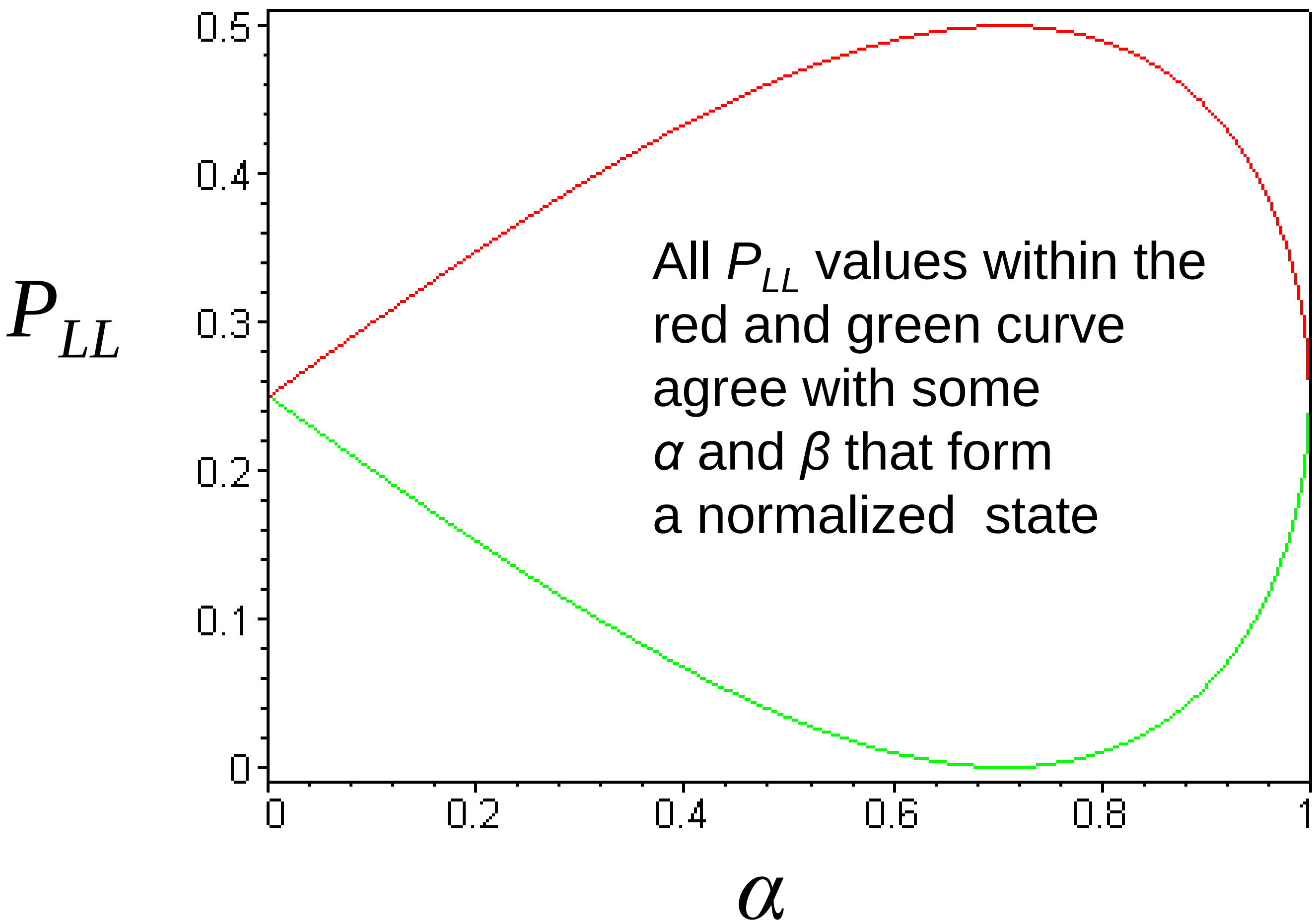
$$\begin{aligned} P_{LL} &= \left| \left(\langle\varphi_{L1}| \langle\varphi_{L2}| \right) |\Psi_T\rangle \right|^2 \\ &= \left| \left(\left(\frac{\langle\varphi_{g1}| + \langle\varphi_{e1}|}{\sqrt{2}} \right) \left(\frac{\langle\varphi_{g2}| + \langle\varphi_{e2}|}{\sqrt{2}} \right) \right) |\Psi_T\rangle \right|^2 \\ &= \tfrac{1}{4} \left| \left(\langle\varphi_{g1}|\langle\varphi_{g2}| + \langle\varphi_{g1}|\langle\varphi_{e2}| + \langle\varphi_{e1}|\langle\varphi_{g2}| + \langle\varphi_{e1}|\langle\varphi_{e2}| \right) \left(\alpha |\varphi_{g1}\rangle|\varphi_{e2}\rangle + \beta |\varphi_{e1}\rangle|\varphi_{g2}\rangle \right) \right|^2 \\ &= \tfrac{1}{4} \left| 0 + \alpha \langle\varphi_{g1}|\langle\varphi_{e2}||\varphi_{g1}\rangle|\varphi_{e2}\rangle + \beta \langle\varphi_{e1}|\langle\varphi_{g2}||\varphi_{e1}\rangle|\varphi_{g2}\rangle + 0 \right|^2 \\ &= \tfrac{1}{4} \left| \alpha + \beta \right|^2 \end{aligned}$$

Related to P_{LL} in this week's handout on exchange and identical particles

We can assume α is real and $\alpha \geqq 0$ and have $|\alpha|^2 + |\beta|^2 = 1$

Any (complex!) β then obeys $\beta = \exp(i\varphi_{\alpha\beta})|\beta| = \exp(i\varphi_{\alpha\beta})\sqrt{1-\alpha^2}$

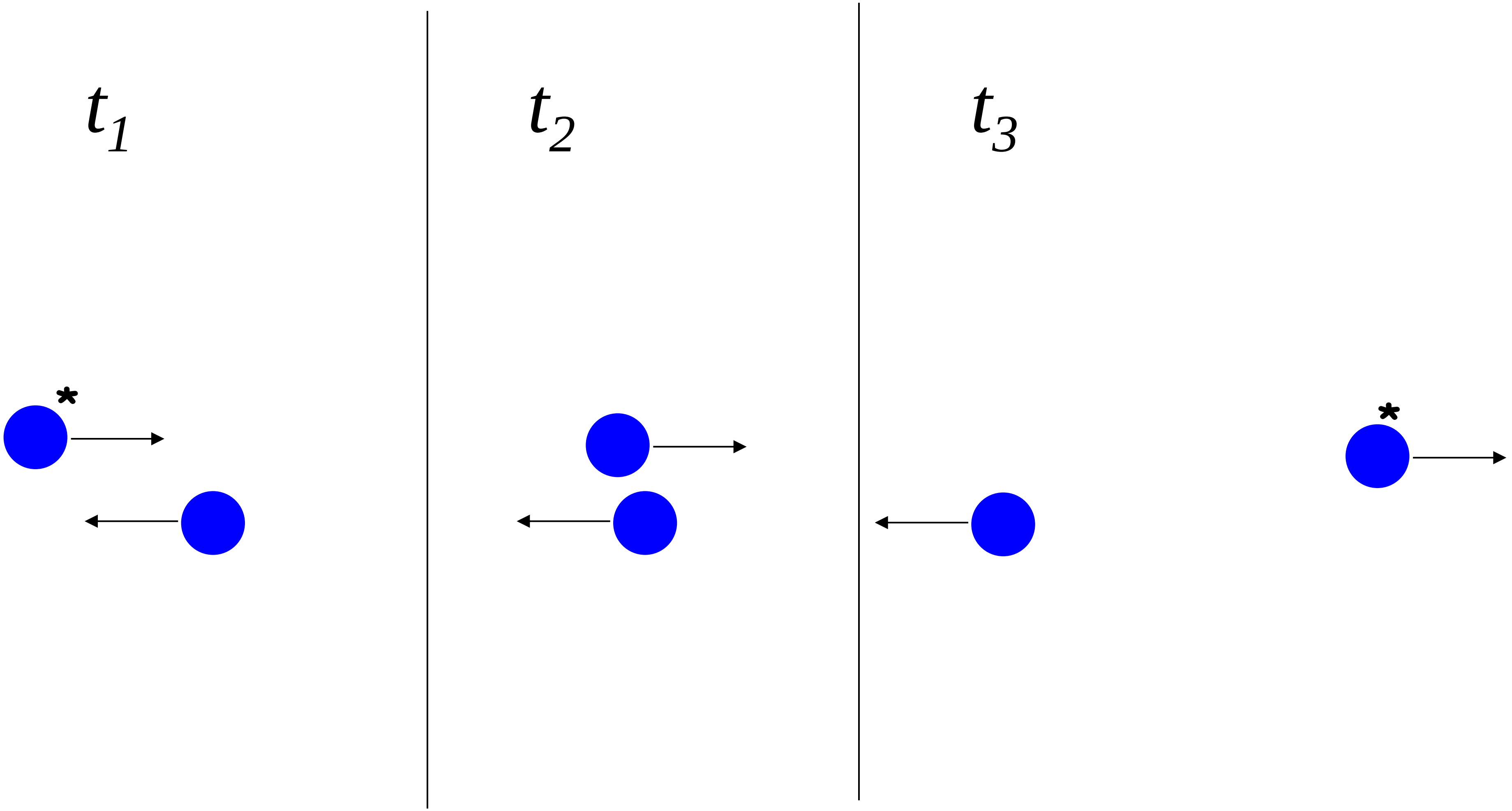
$$P_{LL} = \tfrac{1}{4}\left|\alpha + \exp(i\varphi_{\alpha\beta})\sqrt{1-\alpha^2}\right|^2$$

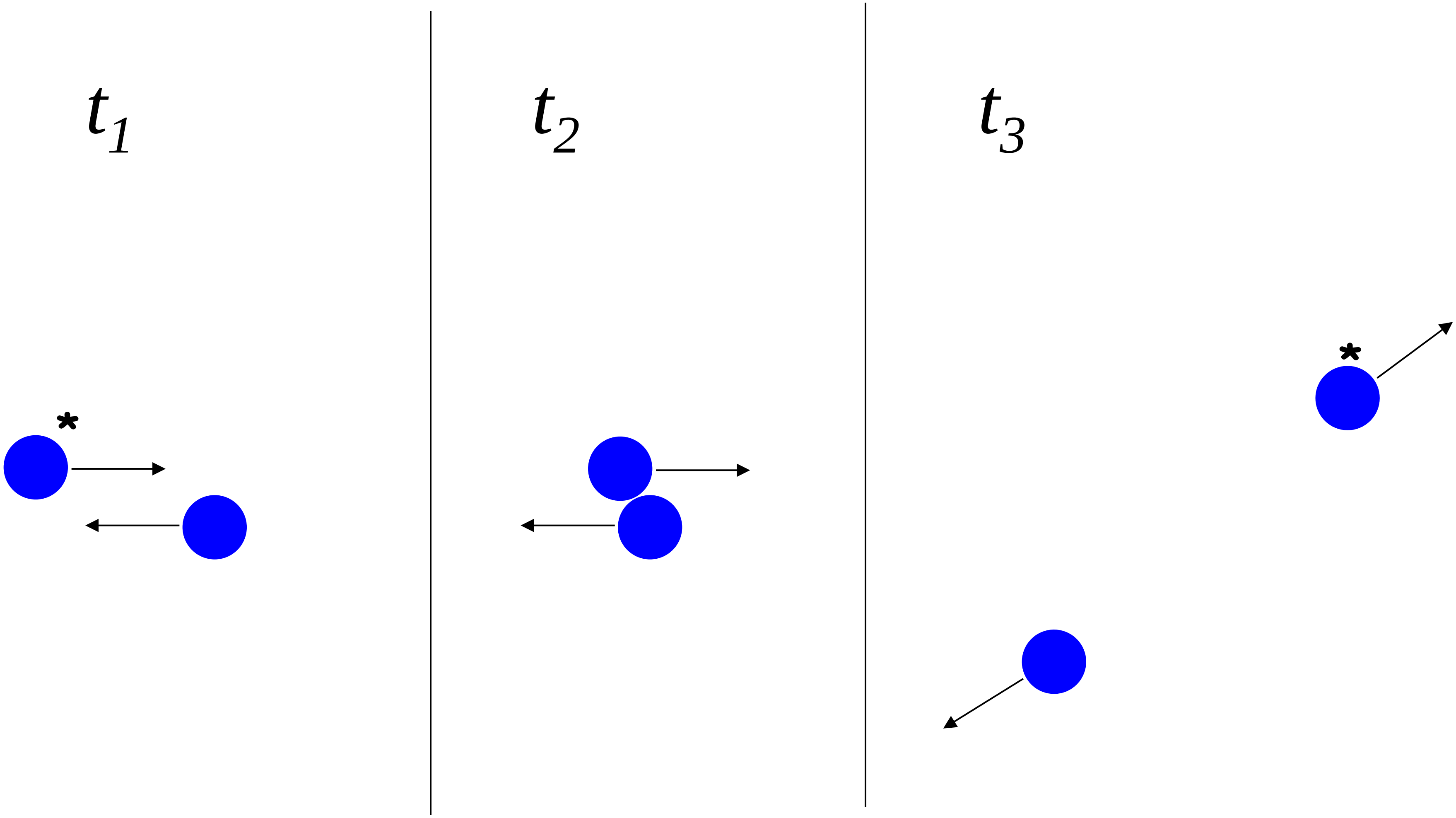


Fermions and bosons

Symmetry, identical particles, and exchange…

Assume a collision experiment with classical particles:

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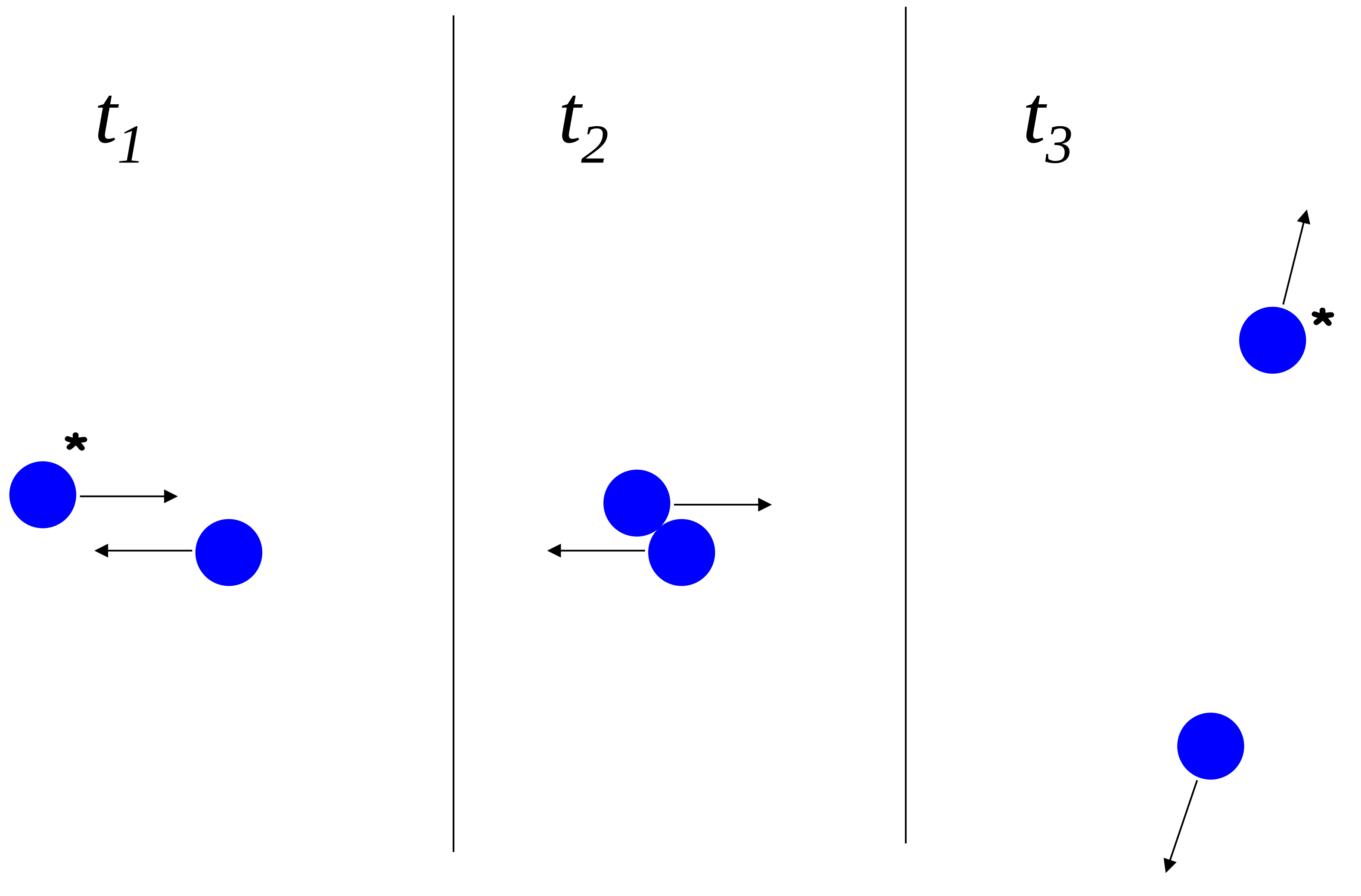
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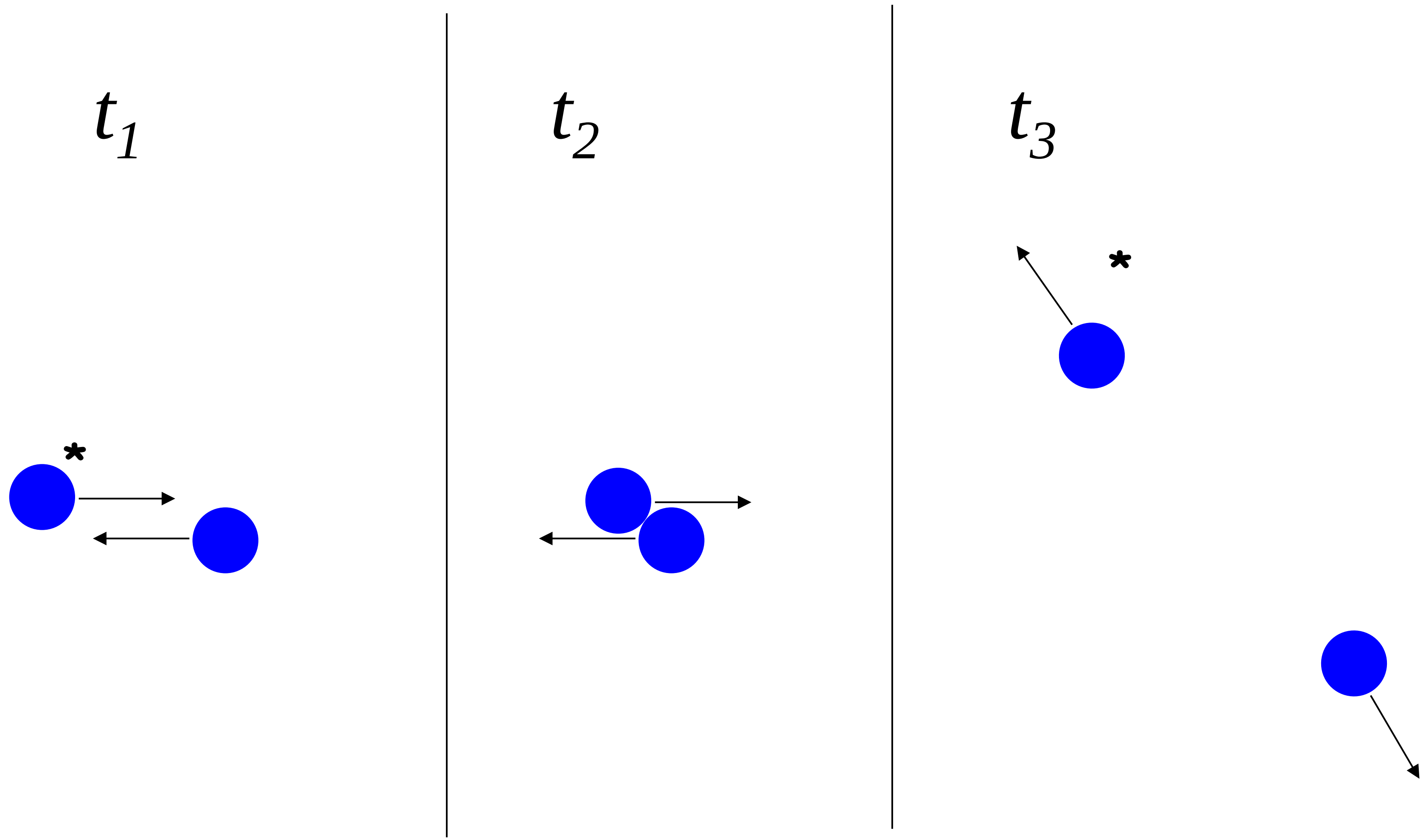
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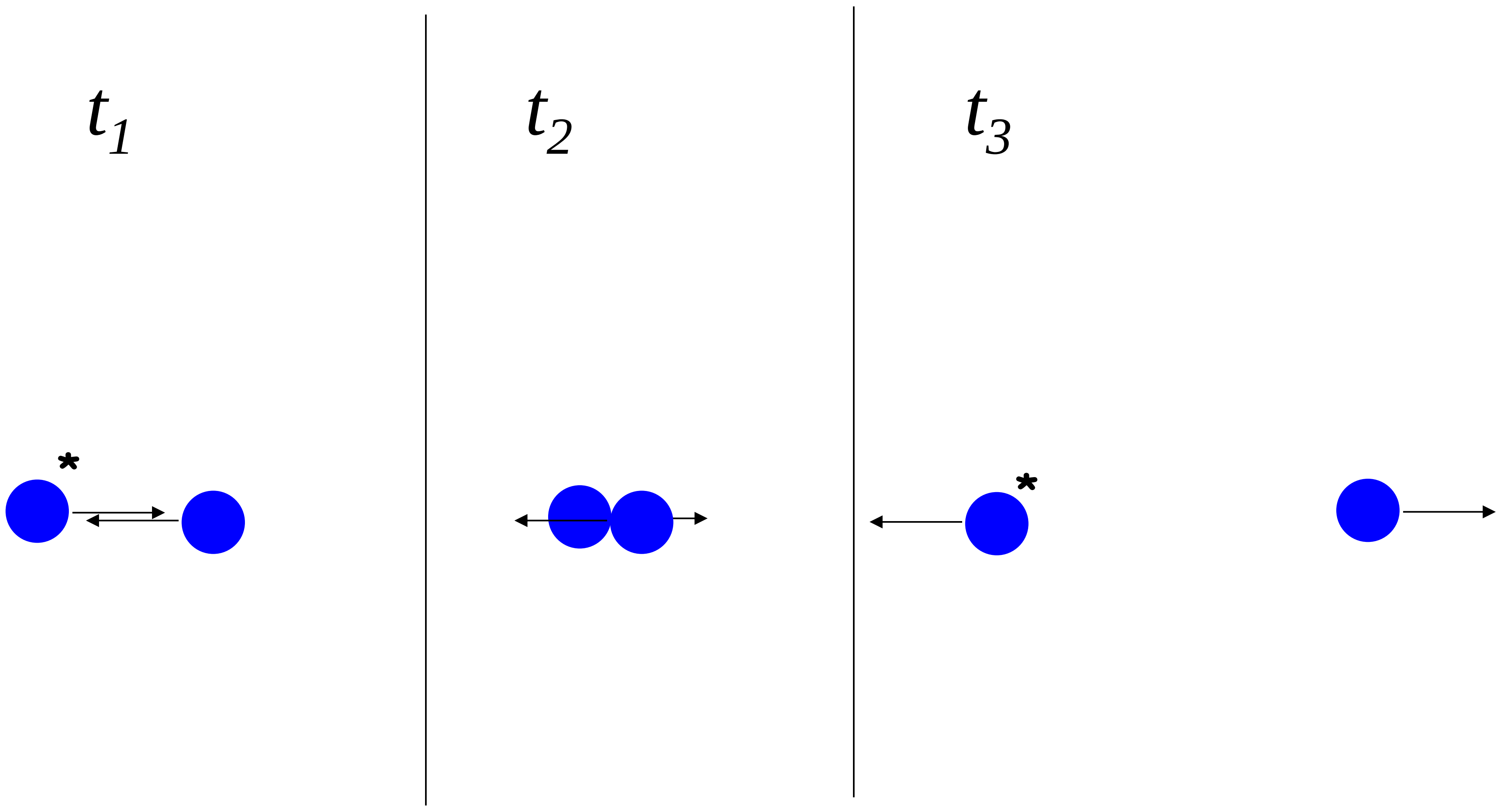
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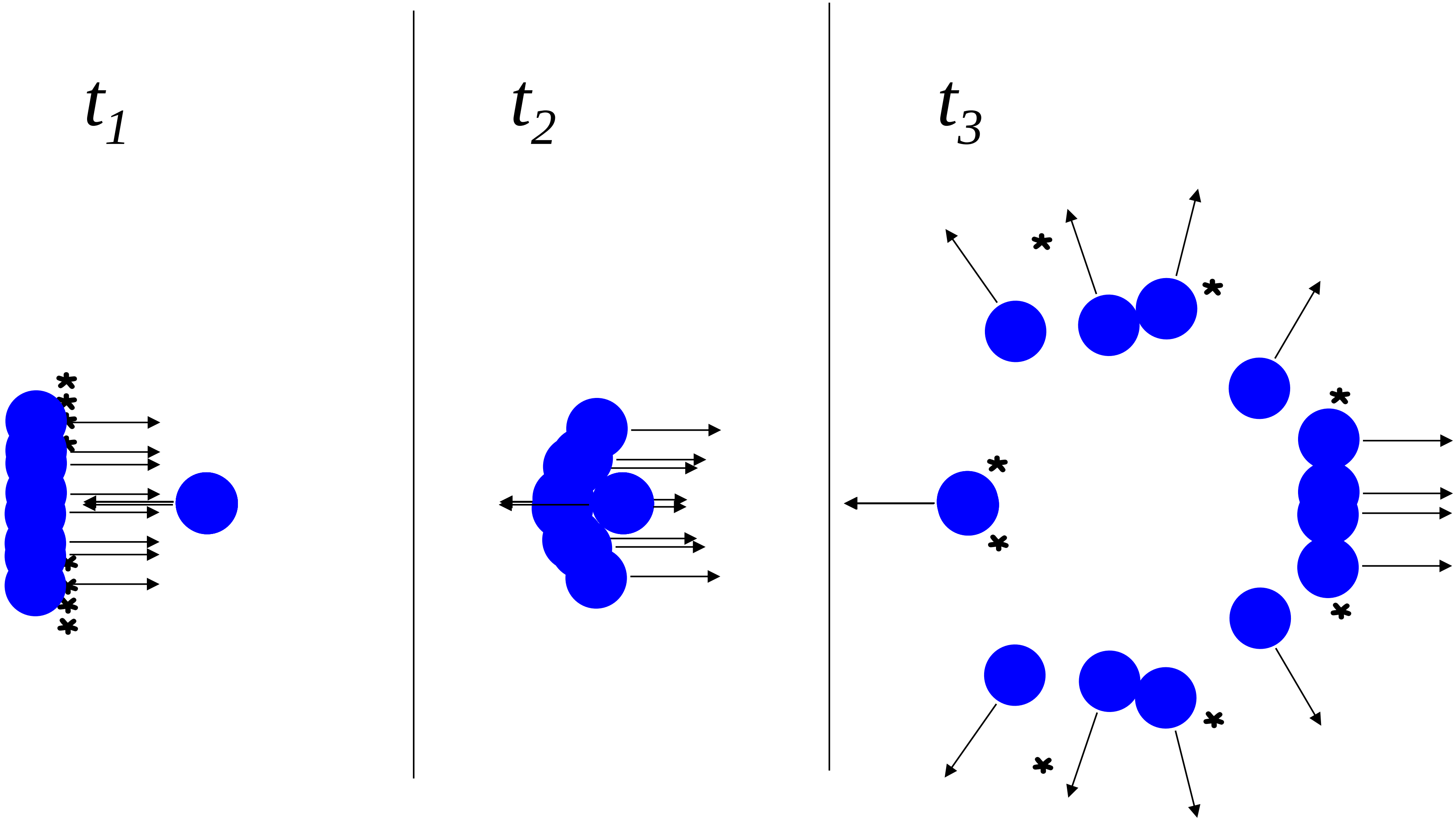
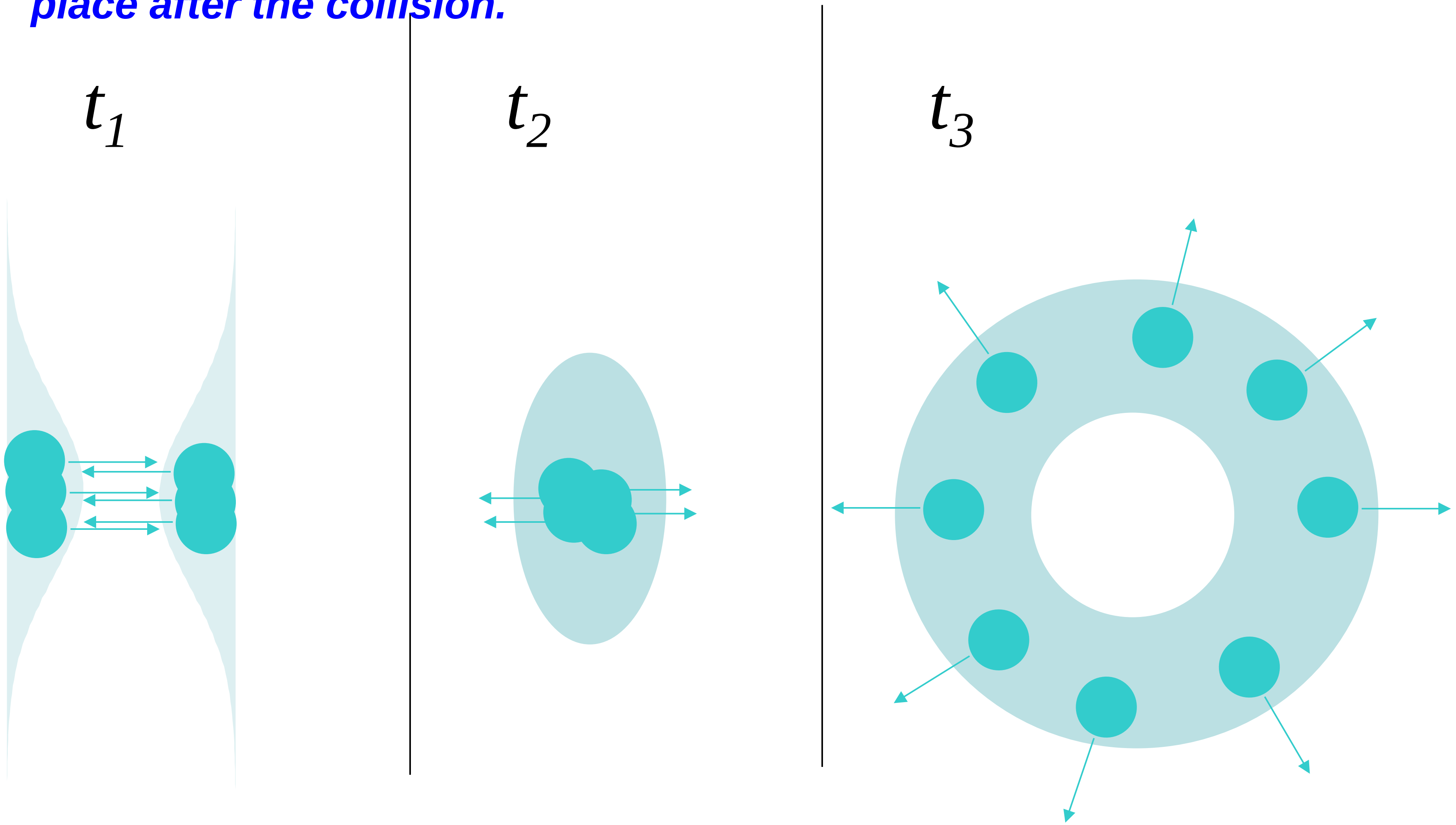


t_1
t_2
t_3

Assume a collision experiment with two <u>IDENTICAL</u> quantum particles:

It is fundamentally impossible to tell which of the two particles is detected, if one detects a particle at some place after the collision.



Symmetry and identical particles

Exchange leads to degenaracy

Symmetry leads to degeneracy

There can be accidental degeneracy

Summary:

Symmetry, systems with two identical particles, degeneracy, exchange