Quantum Physics 1

Lecture of 17 Sept. 2015

Quick check list and mini test for midterm exam

Concepts you must know, be able to apply for the mid-term exam:

- Expectation value $\langle x \rangle$ for a state $\Psi(x)$
- Uncertainty Δx for a state $\Psi(x)$
- Normalizing a state $\Psi(x)$
- Heisenberg uncertainty relation
- Probability for measurement outcomes if you measure x, given a state $\Psi(x)$
- State of a system after measurement
- Dirac notation
- Inner product in Dirac notation
- Expectation value in Dirac notation
- Hamiltonian
- Energy eigenvalues and eigenvectors
- Eigenvectors of an observable are orthogonal
- Expectation value as a function of time for any observable $\hat{A}$
- Time evolution operator $\hat{U} = e^{\frac{-i\hat{H}t}{\hbar}}$ for “kets” and $\hat{U}^{+} = e^{\frac{+i\hat{H}t}{\hbar}}$ for “bra’s”
- Fourier transform between x-representation and k-representation of a state
- Particle in a box (infinitely deep square well)

Write down -by heart- the expression for calculating the Fourier transform from x-representation to k-representation of a state

$$\Psi(x) \quad \overset{\mathcal{F}}{\leftrightarrow} \quad \overline{\Psi}(k)$$

$$\overline{\Psi}(k) =$$

Write down -by heart- the expression for calculating the Fourier transform from x-representation to k-representation of a state

$$\Psi(x) \quad \overset{\mathscr{F}}{\leftrightarrow} \quad \overline{\Psi}(k)$$

$$\overline{\Psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx}\, \Psi(x)\, dx$$

$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx}\, \overline{\Psi}(k)\, dk$$

Quick check:

$\hat{A}$ is an observable (operator) with eigenvalues A_1 and A_2, and associated normalized eigenfunctions $\varphi_1(x)$ and $\varphi_2(x)$.

Say:

$$\Psi = \sqrt{\frac{1}{3}}\varphi_1 + \sqrt{\frac{a}{3}}\varphi_2$$

For what *a* is this state normalized?

2

Note: Can also be done in Dirac notation

Quick check:

$\hat{A}$ is an observable (operator) with eigenvalues A_1 and A_2, and associated eigenfunctions $\varphi_1(x)$ and $\varphi_2(x)$.

Say:

$$\Psi = \sqrt{\frac{1}{3}}\varphi_1 + \sqrt{\frac{2}{3}}\varphi_2$$

What is the probability for measurement outcome A_1?

1/3

Quick check:

$\hat{A}$ is an observable (operator) with eigenvalues A_1=0 and A_2=1, and associated eigenfunctions $\varphi_1(x)$ and $\varphi_2(x)$.

Say:

$$\Psi = \sqrt{\frac{1}{3}}\varphi_1 + i\sqrt{\frac{2}{3}}\varphi_2$$

What is now the probability for measurement outcome A_2?

2/3

Quick check:

$\hat{A}$ is an observable (operator) with eigenvalues $A_1=1$ and $A_2=2$, and associated eigenfunctions $\varphi_1(x)$ and $\varphi_2(x)$.

Say:

$$\Psi = \sqrt{\frac{1}{3}}\varphi_1 + i\sqrt{\frac{2}{3}}\varphi_2$$

What is the value of $\int_{-\infty}^{\infty} \varphi_1^*(x)\hat{A}\varphi_2(x)dx$ **?**

0

Quick check:

$\hat{A}$ is an observable (operator) with eigenvalues $A_1=0$ and $A_2=1$, and associated eigenvectors $|\varphi_1\rangle$ en $|\varphi_2\rangle$.

Say:

$$|\Psi\rangle = \sqrt{\frac{1}{3}}|\varphi_1\rangle + i\sqrt{\frac{2}{3}}|\varphi_2\rangle$$

What is the value of $\langle \hat{A} \rangle$ **?**

$$\langle \hat{A} \rangle = \langle \Psi | \hat{A} | \Psi \rangle$$

2/3