Quantum Physics 1

2015-2016

Teacher: Caspar van der Wal
Tel. 363 4555
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Room NB4 - Gebouw 13 – kamer 140

Organizational announcements

See also the **syllabus** and **http://www.quantumdevices.nl/teaching/**

- Study material for the midterm exam and final exam
- Planning of lectures (hoorcolleges)
- Extra study material
- Tutorials (werkcolleges)
- Bonus from the midterm exam

Homework for the 1st tutorial

(in a few hours, or tomorrow…….!)

Study: Chapter 1

See **http://www.quantumdevices.nl/teaching/**

Problems:

To be made before the tutorial
Chapter 1 - 1.1, 1.2, 1.3, 1.4, 1.7, 1.18

For week 2, see the website

Study material for the midterm and final exam

Book: Introduction to Quantum Mechanics
David J. Grifiths – 2nd edition

List of study material - see the study guide (on the web, updated every week) – roughly:

Midterm: Chap. 1 - part of Chap. 3 with the related problem sets and extra handouts

Final: Chap. 1 - Chap. 5 with the related problem sets and extra handouts

Lectures & Tutorials

See the roster

http://www.rug.nl/fwn/roosters/2015/vakken/nakf1-11

Extra study material

- Study guide (updated each week of the course)
- Extra study material – handouts from the lectures
- Extra exercises (problem sets) – handouts from the tutorials
- Slides of many of the lectures (but part is done on the board)

Available at these points:

- Handed out at the lectures and tutorials.
- On the web **http://www.quantumdevices.nl/teaching/**

  (there is a link from http://nestor.rug.nl/)
- The answers to the problem sets will appear on

  http://nestor.rug.nl/ for this course)
- Table in the hall next to room 140, building 13 of FWN-NB4

How we work during the tutorials I:

- Make sure you come prepared: study the theory in the book work out the problems that are listed as homework.

- Part of the full problem set is more difficult. During the tutorials you should be working on these more difficult problems.

- The only plenary teaching during the tutorials are short summaries of this week’s topics, a few minutes at the start and/or end of a class.

- **There is no plenary teaching on the problem solving.** It is our intention that you spend most of your time on working out the problems yourself. The instructor is only there to get you forward again if you would get stuck.

How we work during the tutorials II:

- We encourage that you work on the problems together with your fellow students.

- Answers and worked-out problems for many of the problems will appear on **nestor** at the end of the last tutorial session of a week.

- Do you have questions? It is **your responsibility** to approach the instructors during or after a tutorial session of a lecture**!**

Bonus test = Midterm exam

On Thursday 24 Sept. 2015, 09:00-10:00
in the Aletta Jacobs Exam Hall

The rules of the game:

- The grade on the mid-term exam is an integer number.
- Only if your mid-term grade is $\geqq$ 6, your final grade for this course will be: (grade final exam) + (grade mid-term exam)/10.
- The final grade on the course cannot be higher than 10.
- This bonus regulation is only valid in combination with the first final exam after the end of the lectures of this year.
- Participating in the mid-term exam and this bonus regulation is not obligatory.

Indication of the required study load

This course is 5 ECTS = 140 hours

- 8 weeks with ~8 contact hours per week
- Homework per week: 2 x 1 hour study/reading theory
  2 x 2 hours working out problem sets
- Further preparing for the exams ~28 hours

Homework

Homework for the tutorials on Wednesday and Friday will be announced each week on Tuesday:

Theory:	Study material (book and extra handouts)
	Reading material
	Contents of lectures and slides
Problem sets:	Part is homework
	Part is for during the tutorial session
	Extra problems (optional material)

Bonus-test Thursday 24 September 2015, 9:00-10:00

Final exams See the roster on the web
(2 times per year, in November & few weeks later).

End organizational announcements.

Questions?

Quantum Physics 1

2015-2016

Lectures 1 and 2 September 2015

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Why QUANTUM MECHANICS?

About 120 years ago, there were a range of experimental results that could not be explained with classical theorys.

For example: Spectral lines of atoms.
Interference of electron beams.

Today no examples in detail, several will come in later lectures or in later chapters of the book.

Why is quantum physics important?

- Why are gold and copper good electrical conductors, while glass is not?

- Why is red paint red, and blue paint blue?

Why is quantum physics important?

- Atomic en molecular physics
- Solid-state physics
- High energy physics
- Physical chemistry
- Quantum information technology

The difficicult part of the course is not the mathematics, but that theory is so counter intuitive $\Rightarrow$

- Very nice course!
- Do you reading in advance, and come with questions!

QUANTUM MECHANICS

The essential differences between classical mechanic and quantum mechanics concerns:

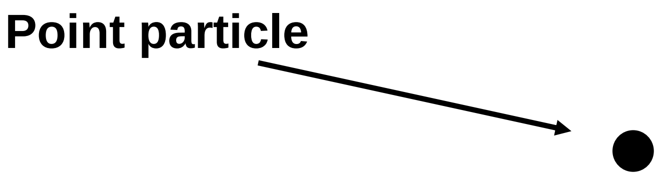
→ **1) The state of a physical system**

2) The time evolution of a physical system

3) Making measurements on a physical system

1) The state of a point particle – classical

How was it again for classical mechanics?



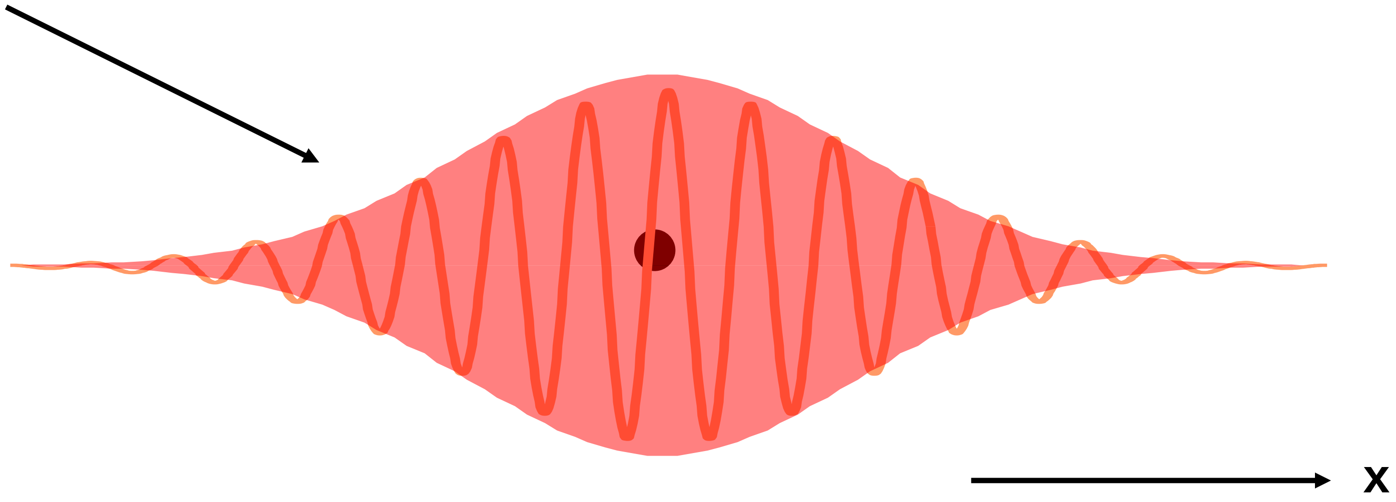
We know everything if we know the x-, y-, z-position and the p_x-, p_y- en p_z-momentum

$\Rightarrow$

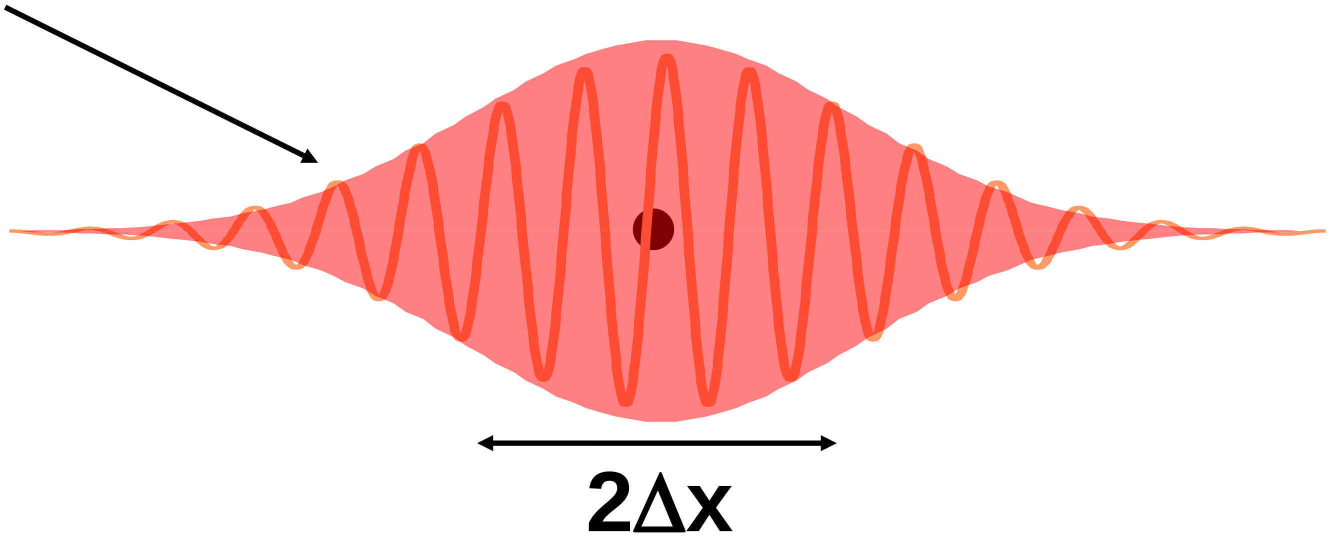
6 numbers with a specific value

The state of a point particle – quantum version

Wave function (complex!) describes the x-position of a point particle



The state of a point particle:
the wave describes the x-position of a point particle.



Also the linear momentum (in x-, y- and z-direction) is described by a <u>wave function</u>.

Moreover, it turns out that it is impossible to realize a state that has at the same time very little spread in x-position and x-momentum (Heisenberg uncertainty relation).

$$\Delta x \cdot \Delta p_x \geq \hbar / 2 \qquad \left(\hbar = h / 2\pi\right)$$

Note on notation: Δx vs σ_x and Δp_x vs σ_{p_x}

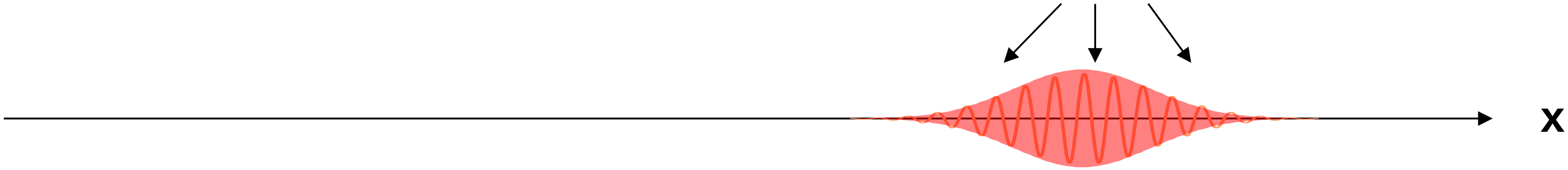
For the notation of the quantum uncertainties on the previous slide, very many (most?) books and publications use Δx and Δp_x .
We will also often use this in this course.

The book by Griffiths uses here σ_x and σ_{p_x} instead.

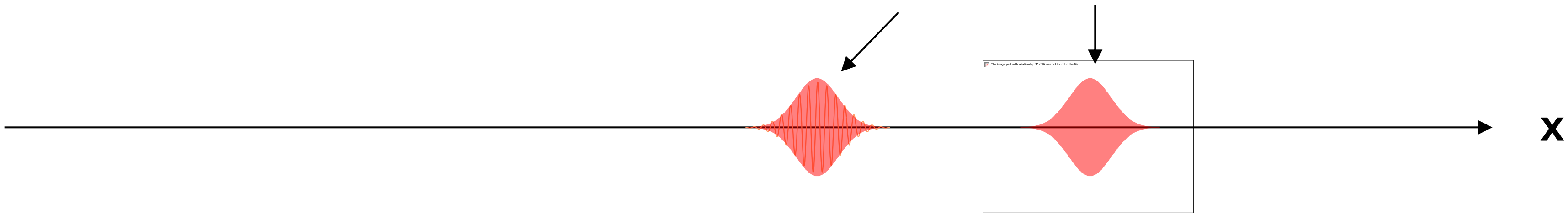
Consequence: superposition principle

Classical – particle is exactly here

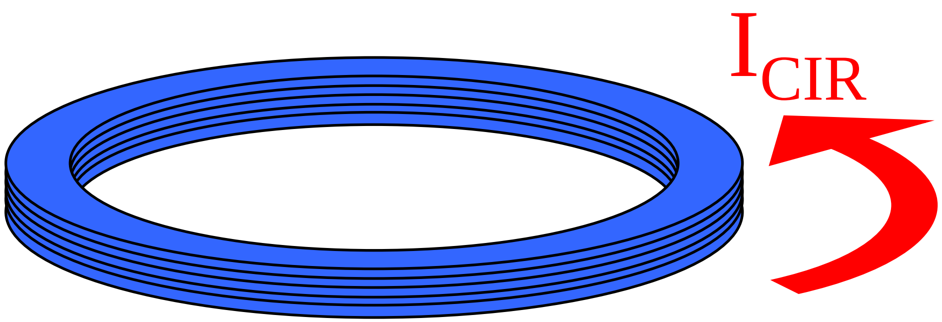
Quantum – particle is at all these places at the same time…

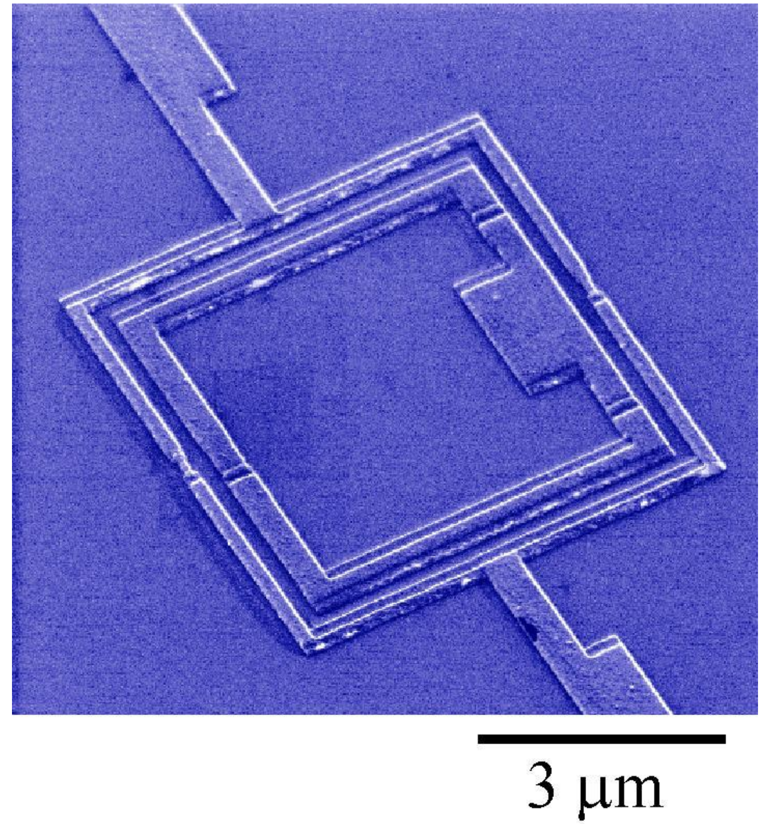


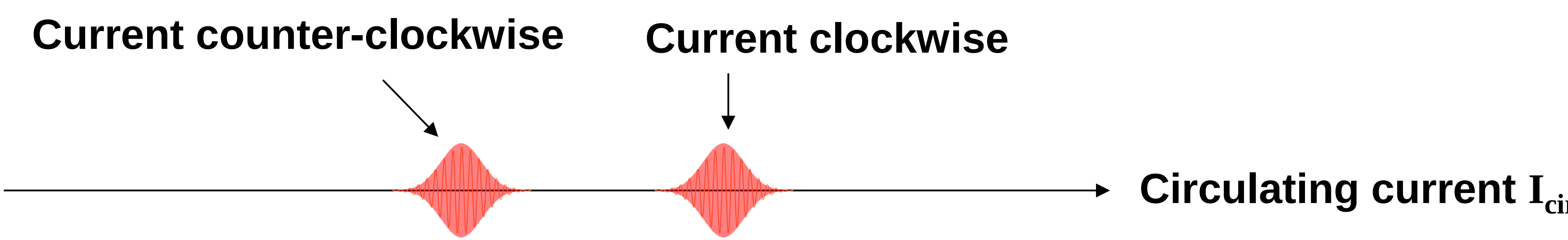
…. or even in two separated areas at the same time!



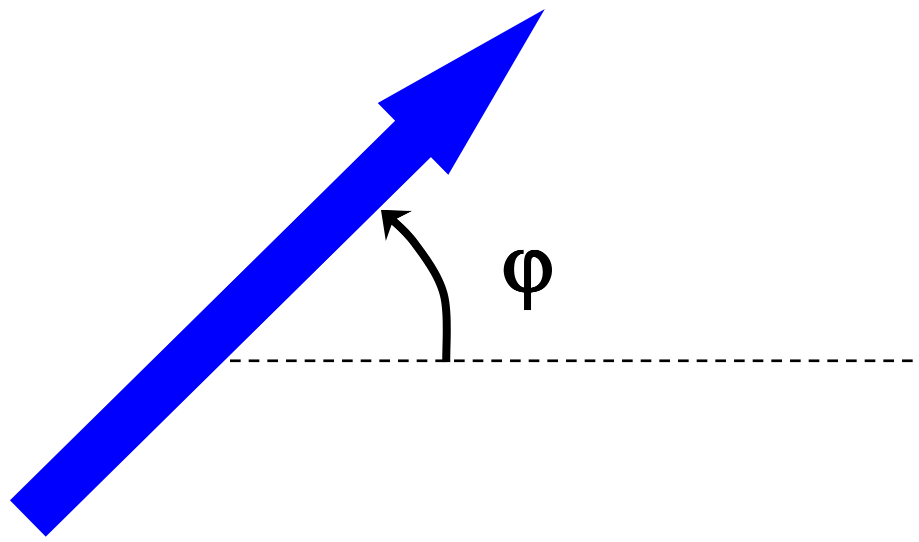
……and it does not have to be about the position of a point particle:



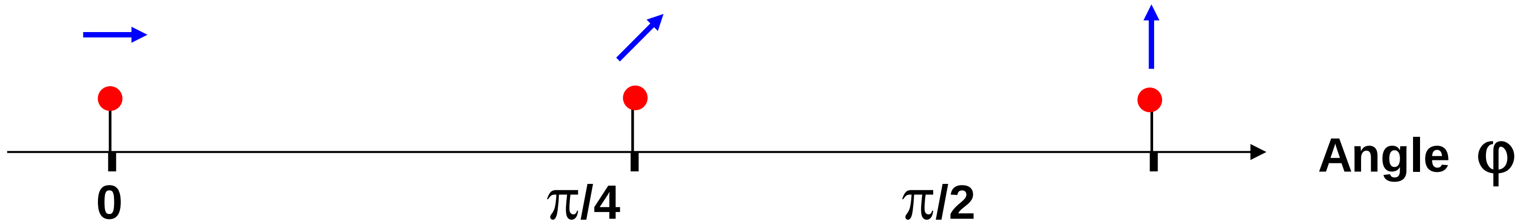




One more example: the direction of an arrow (small magnet)



Superposition of 3 discrete states

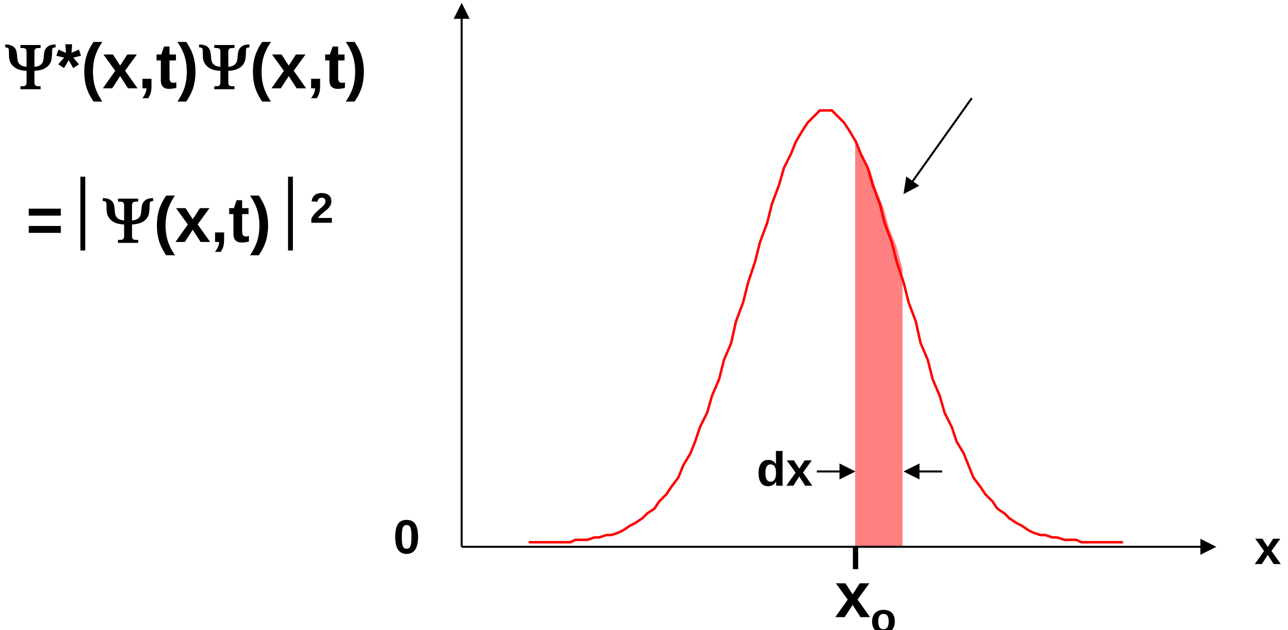


Interpretation van the wave function: probability density

Say, for some time *t*, the state of a degree of freedom x is described by the complex wave function $\Psi(x,t)$:

The probability P for x to be in the small range around x_o of width dx is
$P(x_o < x < x_o + dx) = |\Psi(x_o,t)|^2 dx$

So, there is a **probability density**
$W(x) = |\Psi(x,t)|^2$



Complex conjugate (you should know this….!)

$$(x+iy)^* = x - iy \qquad \text{or}$$

$$(a+ib)^* = a - ib \qquad \text{with} \quad i^2 = -1$$

Which we just applied as:

$$(x+iy)\times(x+iy)^* = (x+iy)^* \times (x+iy)$$
$$= |x+iy|^2$$
$$= x^2 + y^2$$

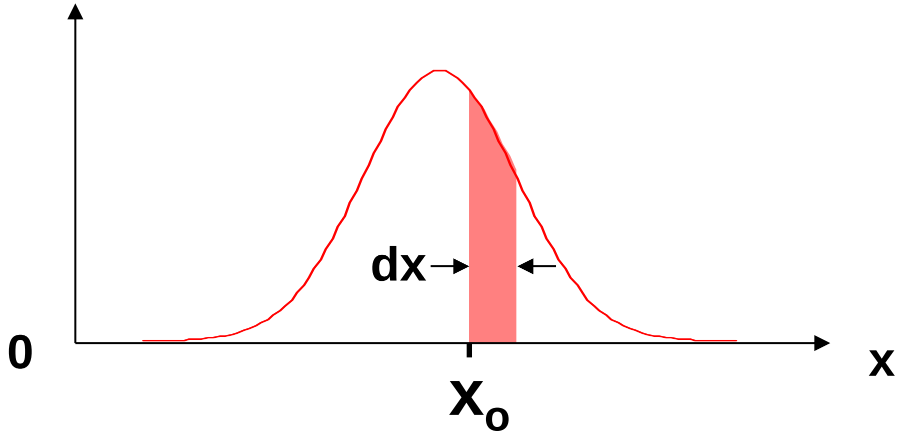
Interpretation van the wave function: probability density

The particle is always somewhere, so

$$\int_{-\infty}^{\infty} \left|\Psi(x,t)\right|^2 dx = 1$$

$$\Psi^*(x,t)\Psi(x,t)$$

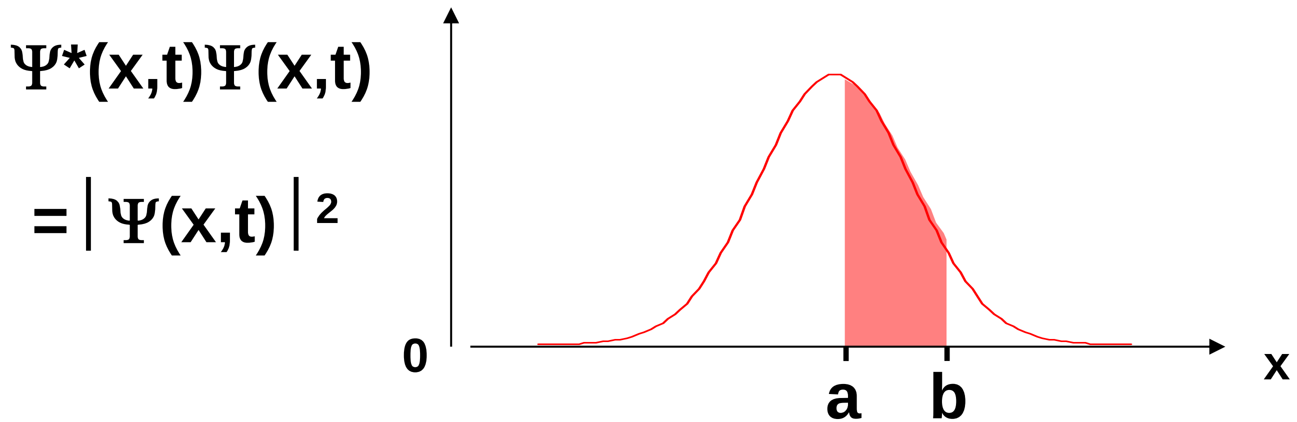
$$= \left|\Psi(x,t)\right|^2$$



Interpretation of the wavefunction: probability

What is the probability for the particle being/finding it between $x = a$ and $x = b$ (for example when you would measure where it is)

$$P(a < x < b) = \int_a^b |\Psi(x, t_0)|^2 dx$$



Normalizing the wave function

As the physical solution of a differential equation, Ψ(x,t) is often known besides a multiplicative constant *C*:

For example, for a certain *t*, you found the mathematical solution *C*Φ(x,t) :

$$\Psi(x,t) = C\,\Phi(x,t) \quad \Rightarrow$$

$$\int_{-\infty}^{\infty} \left|\Phi(x,t)\right|^2 dx = A \quad \Rightarrow$$

$$C = \frac{1}{\sqrt{A}}, \qquad \Psi(x,t) = \frac{1}{\sqrt{A}}\Phi(x,t)$$

This you need for your homework this week:

<u>Expectation value</u> for the property position x at $t = t_0$

Say, the system is in state $\Psi(x,t_0)$

$$\langle x \rangle = \int_{-\infty}^{\infty} \Psi(x,t_0)^* \; x \; \Psi(x,t_0) dx$$

In the same manner you can calculate the
<u>quantum uncertainty in position x</u> $\Rightarrow$ **Δx**

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \Psi^*(x) \; \hat{x}^2 \; \Psi(x) \, dx = \int_{-\infty}^{\infty} \Psi^*(x) \; x^2 \; \Psi(x) \, dx$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

Note: see the earlier slide on Δx vs σ_x notation.

Do not confuse these three concepts:

Probability density *W(x)*

$$P\left(x_0 < x < x_0 + dx\right) = \left|\Psi\left(x, t_0\right)\right|^2 dx \qquad \Rightarrow$$

$$W\left(x\right) = \left|\Psi\left(x, t_0\right)\right|^2 = \Psi\left(x, t_0\right)^* \cdot \Psi\left(x, t_0\right)$$

Probability *P* for *x* between *a* and *b*

$$P\left(a < x < b\right) = \int_a^b \left|\Psi\left(x, t_0\right)\right|^2 dx$$

Expectation value for *x*

$$\left\langle x \right\rangle = \int_{-\infty}^{\infty} \Psi\left(x, t_0\right)^* \; x \; \Psi\left(x, t_0\right) dx$$

QUANTUM MECHANICS

The essential differences between classical mechanic and quantum mechanics concerns:

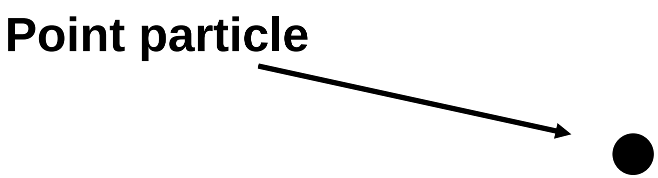
1) The state of a physical system

➡ 2) The time evolution of a physical system

3) Making measurements on a physical system

2) The time evolution of a physical system - Classical

How was it again for classical mechanics?



$$-\frac{\partial V}{\partial x} = m\frac{d^2x}{dt^2}$$

“F=m•a”
A differential equation that precisely describes the time evolution for a specific value of position.
Energy determines the dynamics.

The time evolution of a physical system – quantum version

Say, the state of a quantum system is described by a wave function $\Psi(x,t)$.

Then, the time evolution of this system is described by a differential equation, which is a wave equation, and has the name the Schrödinger equation.

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = \hat{H}\,\Psi(x,t)$$

Hamiltonian, total energy

For an isolated system that is at a certain moment in a well-defined wave function, this dynamics is deterministic!

QUANTUM MECHANICS

The essential differences between classical mechanic and quantum mechanics concerns:

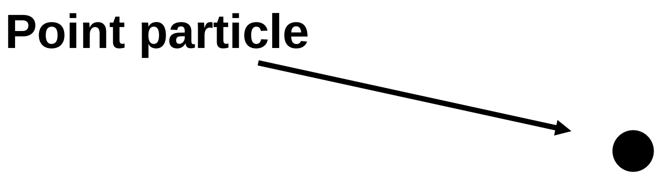
1) The state of a physical system

2) The time evolution of a physical system

→ 3) Making measurements on a physical system

3) Measuring on a physical system - classical

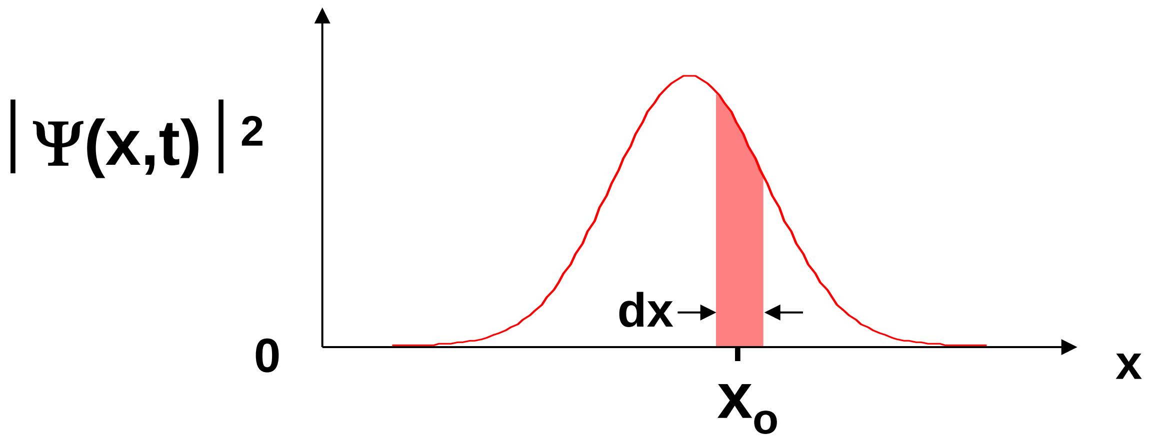
How precisely can we measure x en p_x of a classical point particle?



- Classically there is no fundamental limit for how precisely we can determine x and p_x.
- We can measure x and p_x very precisely at the same time.

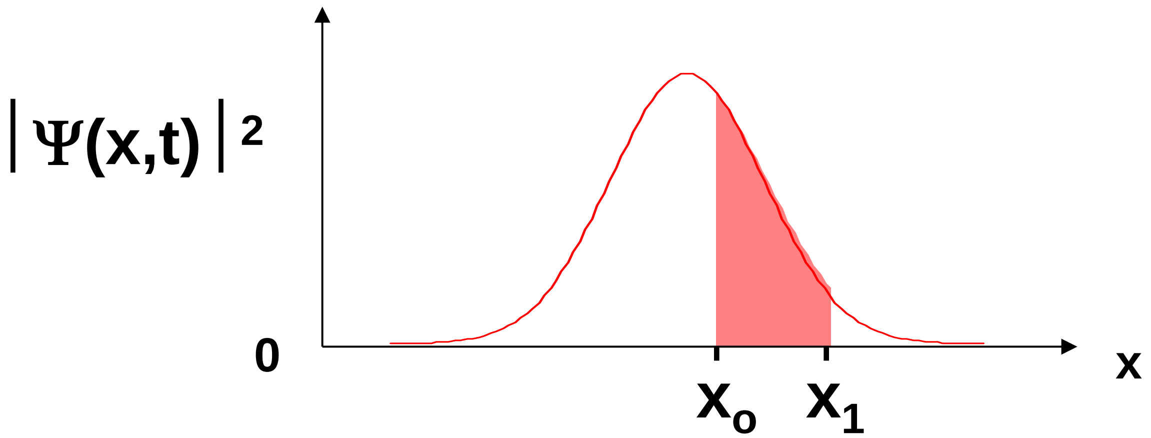
3) Measuring on a physical system – quantum version

How does a measurement work in the quantum world?



The probability for a measurement outcome in the range $x_o \pm dx$ is $|\Psi(x_o,t)|^2 dx$.

Detection whether a particle is in interval from x_o to x_1:

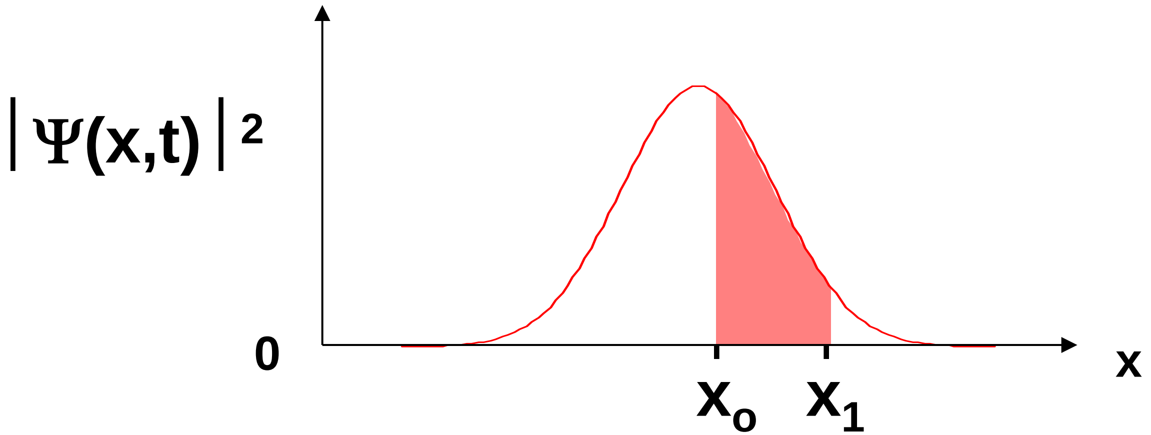


The probability for a measurement outcome in the range from x_o to x_1 is

$$P(x_0 < x < x_1) = \int_{x_0}^{x_1} |\Psi(x,t)|^2 \, dx$$

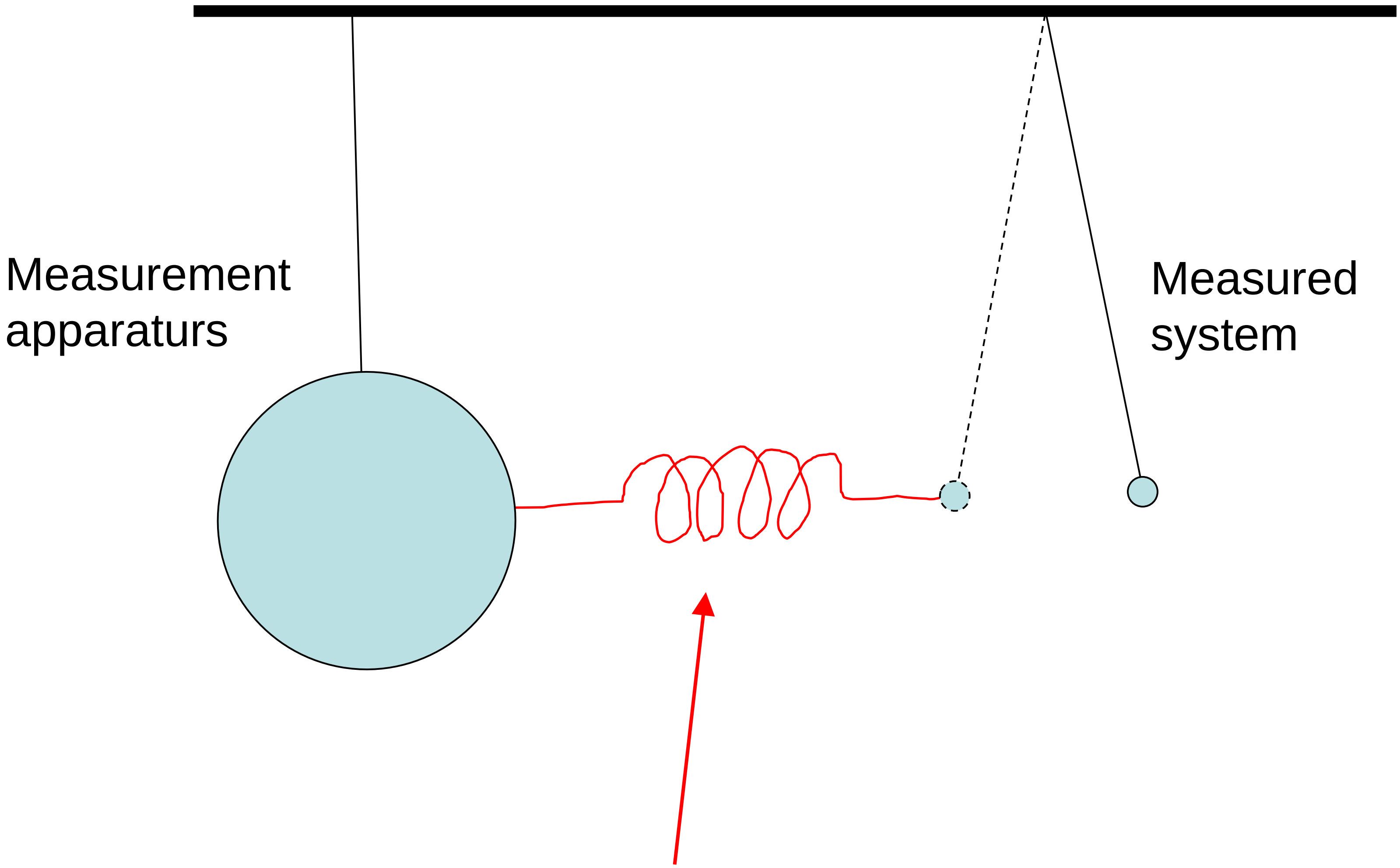
3) Measuring on a physical system – quantum version

How does a measurement work in the quantum world?



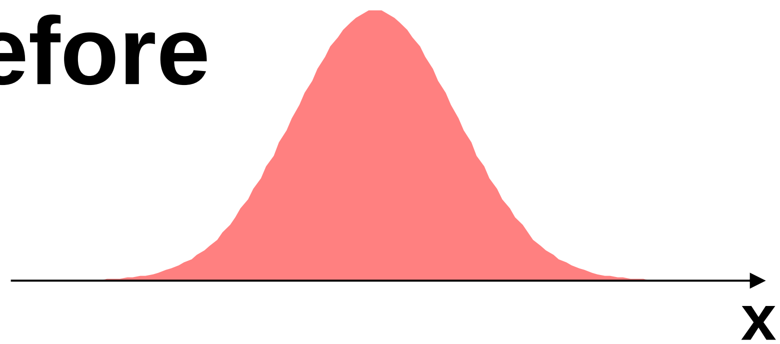
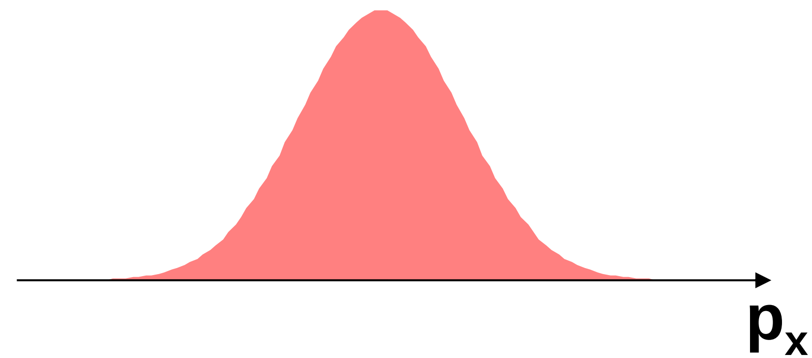
The state after the measurement is (in most cases) strongly disturbed by the measurement process.

Semi-classical cartoon for showing that measuring must result in disturbing the measured system

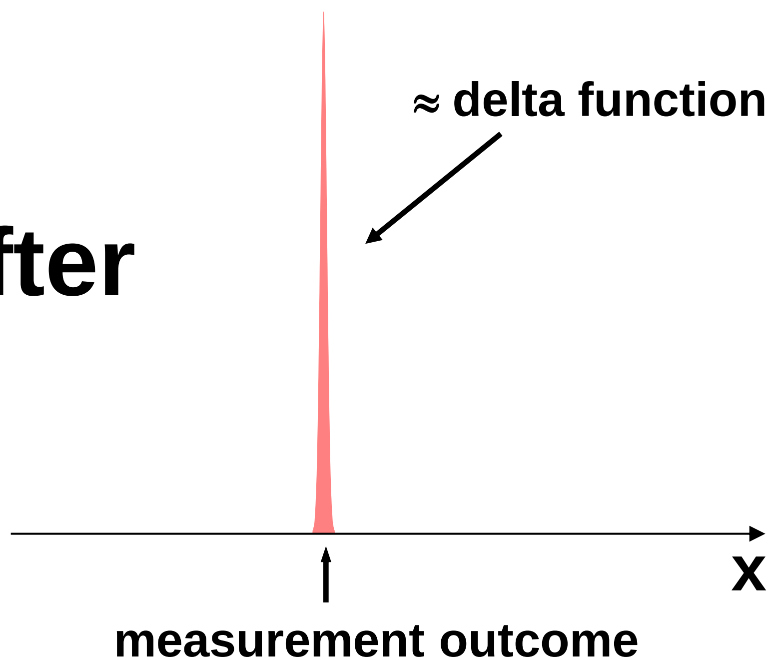


Switching on the measurement is realized by adding this elastic spring connection

Measuring x ⇒ interaction between measurement apparatus and x

	$\mathbf{x}$	$\mathbf{p_x}$
Before		

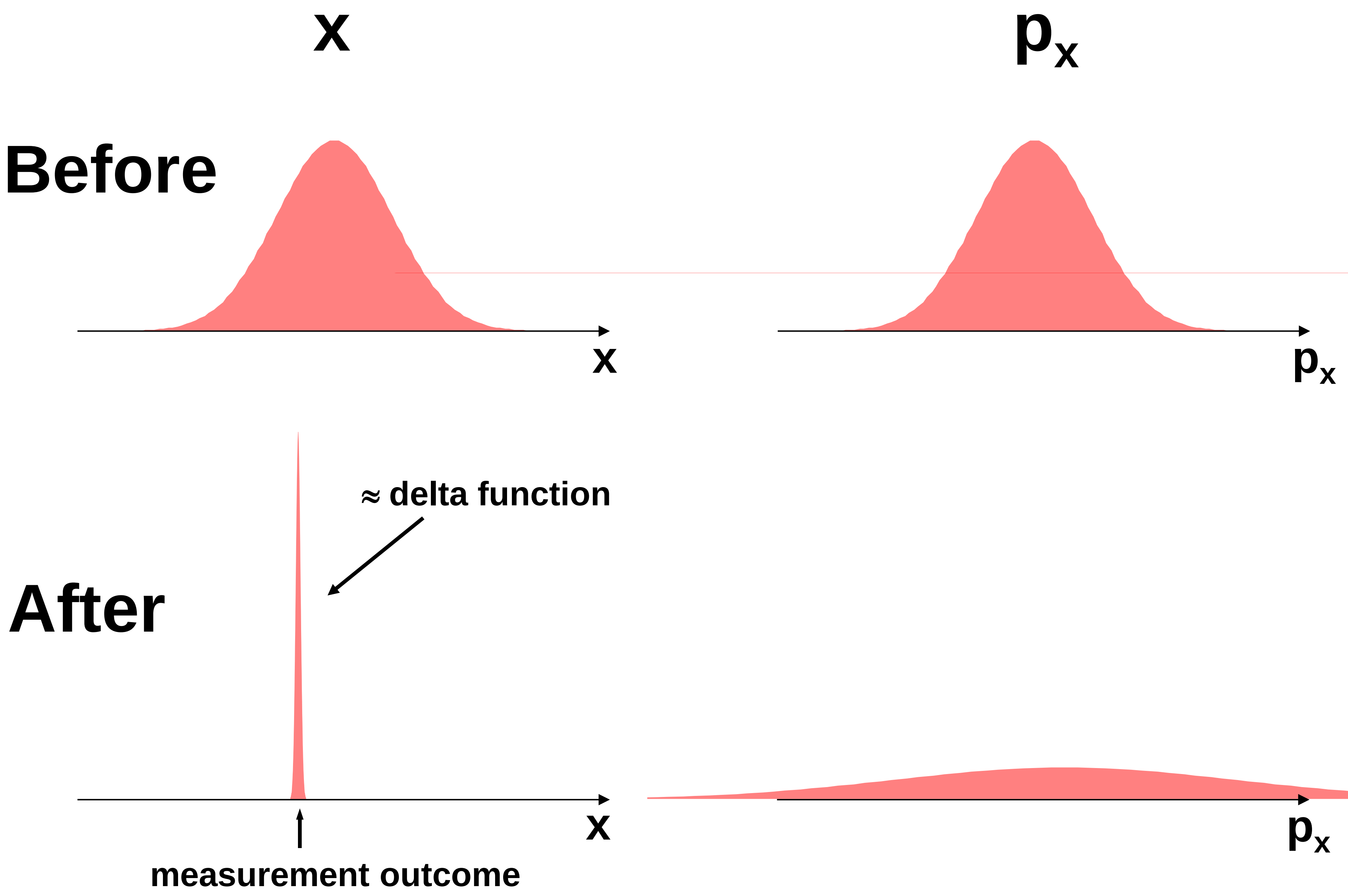
After



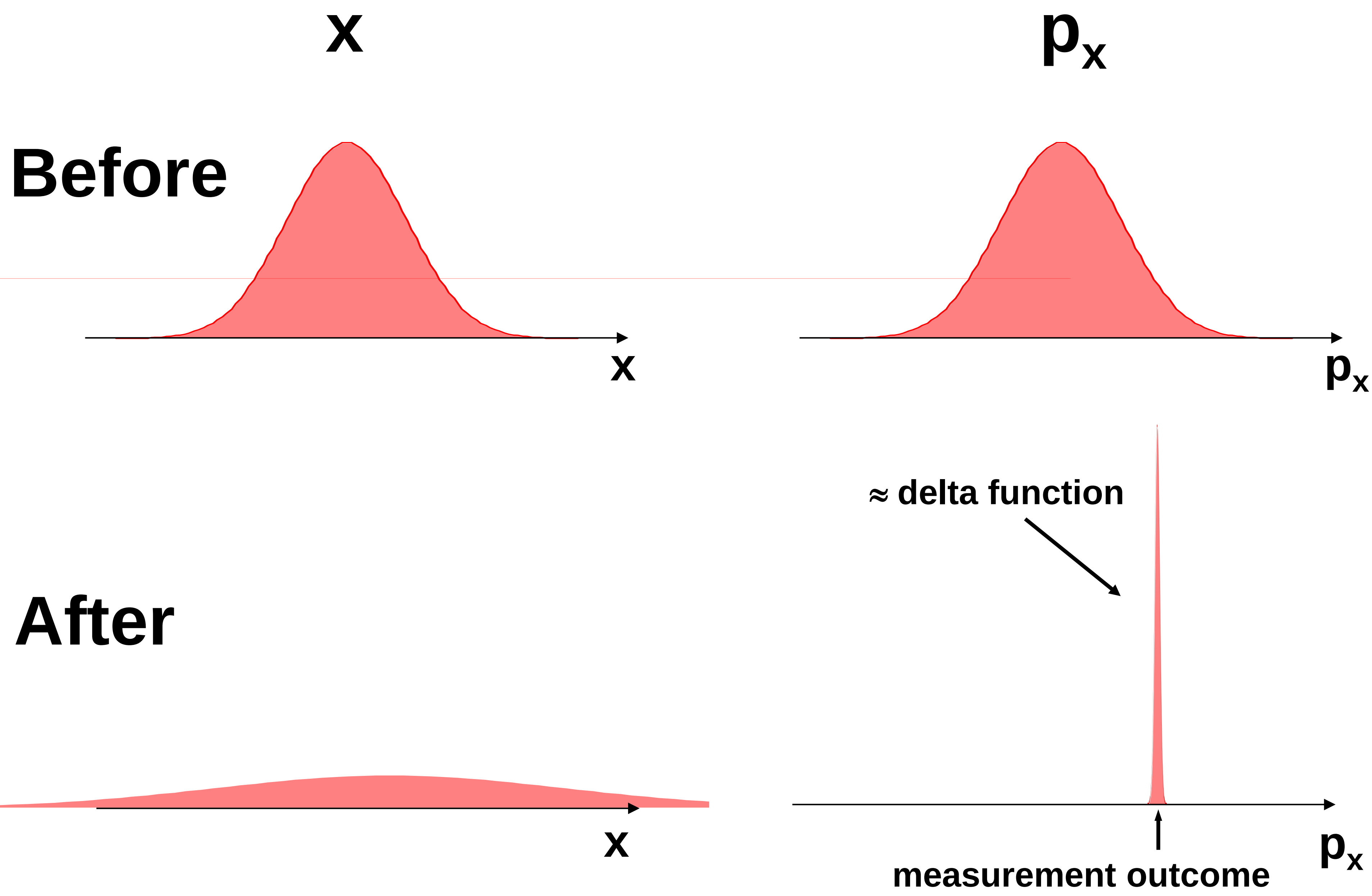
What happens with the wave function that describes $\mathbf{p_x}$?

$$\Delta x \cdot \Delta p_x \geq \hbar / 2$$

Measuring x ⇒ interaction between measurement apparatus and x



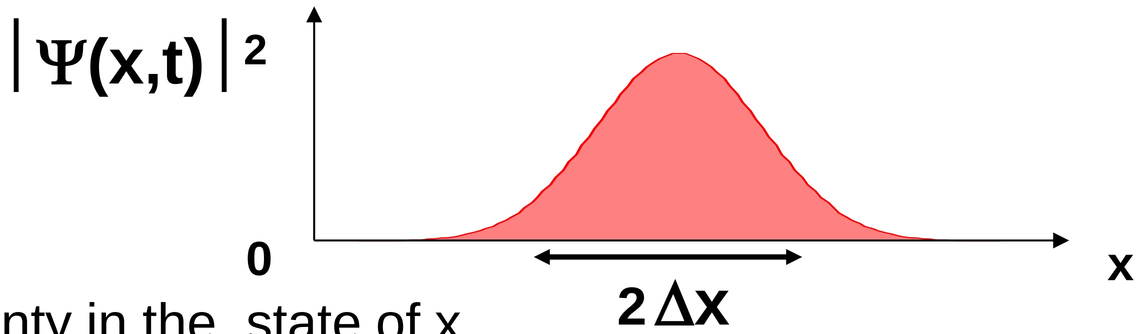
Measuring $p_x \Rightarrow$ interaction between measurement apparatus and p_x



Consequences:

1) We can**not** measure x en p_x as precisely as possible simultaneously.

2) Moreover:



quantum uncertainty in the state of x
already causes an uncertainty for the outcome at each measurement.

3) The order of measuring x and p_x (one or the other first) makes a difference for the outcomes!

4) We do not always know in advance whether Δx or in fact Δp_x is very large, in in these cases one cannot decide very well whether it is better to measure first x or p_x.

5) Repeatedly measuring on identical systems can give very precise results.

Summary:

Quantum mechanics is (next to classical mechanics) a theory for describing the state and time evolution of a physical system:

1. The state is described by a wave function.
2. The time evolution is describe by the Schrödinger equation, a differential equation.
3. Measuring on a quantum system always goes along with a very significant disturbance of the state of the systems, and for outcomes there is a fundamental uncertainty.

Next lectures:

The 5 POSTULATES

Interference of quantum waves in double-slit experiments.

Homework for week 2 of the course

Study: Chapter 2, up to and including 2.3

(part of 2.3 is reading material)

The handout from the Feynman lectures

See **http://www.quantumdevices.nl/teaching/**

Problems:

To be made <u>before</u> the tutorial

Chapter 2 - 2.1, 2.3, 2.4, 2.8, and 2.9