# Problem set for Chapters 8 and 9 for werkcollege Kwantumfysica 1

**Study year 2010-2011, see also werkcollege schedule with planning per group**

This week, continue to work on previous week's problem set for Chapters 7 and 8 if you need more time for it, and then continue with C8-9.1 - C8-9.4 (below).

Then make from the Chapter 9 in the book: 9.15.

(Homework was from Chapter 9 problems: 1, 2, 3, 5, 7a.)

**Problem C8-9.1**

If an electron is in a circular orbit around a nucleus, this state has an angular momentum $\boldsymbol{L}$ and the circular motion of the electron also causes a magnetic moment $\boldsymbol{\mu}$ (at least when considering it semi-classically, quantum mechanically these quantities can be zero for certain states). These two quantities are proportional to each other, according to $\boldsymbol{\mu} = \gamma\, \boldsymbol{L}$, where $\gamma$ is the so-called gyromagnetic ratio. The purpose of this exercise is to obtain the value of $\gamma = e/2m_e$ from a semi-classical calculation, where $e$ the electron charge and $m_e$ the electron mass.

It turns out, that you do not need to know details of the electron orbit. Simply assume that the electron is on a circular orbit of radius $r$ with velocity $v$. Further use that the magnetic moment of a circulation current $I$ in a loop of area $A$ is $\mu = I\,A$. Use this to calculate $\gamma$.

**Problem C8-9.2**

A certain atom is in a state with its total orbital angular momentum $\boldsymbol{L}$ (described by the operator $\hat{L}$) defined by orbital quantum number $l = 1$ (see p. 352 and p. 362 in the book).

**a)** What is the length of the corresponding angular momentum vector $\boldsymbol{L}$ ?

**b)** What are the possible measurement outcomes (for some arbitrary state for which $l = 1$) when measuring the $x$-component of $\boldsymbol{L}$ ?

**c)** For this case with $l = 1$, the complete set of three eigenvalues and eigenstates of the operator for the $z$-component of $\boldsymbol{L}$ is characterized by the following equations:

$$\hat{L}_z|\uparrow\rangle = +\hbar|\uparrow\rangle, \qquad \hat{L}_z|\rightarrow\rangle = 0|\rightarrow\rangle, \qquad \hat{L}_z|\downarrow\rangle = -\hbar|\downarrow\rangle.$$

Instead of using the above Dirac notation, rewrite these three equations in a matrix (3 by 3) and column vector notation (with 3 elements), using the basis that is spanned by $|\uparrow\rangle, |\rightarrow\rangle$ and $|\downarrow\rangle$.

**d)** At some point this system is in the state $|\Psi\rangle = \sqrt{\tfrac{1}{8}}\,|\uparrow\rangle + \sqrt{\tfrac{3}{8}}\,|\rightarrow\rangle + \sqrt{\tfrac{4}{8}}\,|\downarrow\rangle$. Calculate the expectation value $\langle \hat{L}_z \rangle$ for this state. Do it once using Dirac notation, and once using the matrix notation.

**e)** For the state in **d)**, what is the probability to obtain the result $L_z = -\hbar$ when measuring the $z$-component of $\boldsymbol{L}$ ?

**f)** The derivation in the book on p. 360 uses $\langle \hat{L}_z^2 \rangle$ (but there it is written for an angular momentum $\boldsymbol{J}$ ). Calculate $\langle \hat{L}_z^2 \rangle$ for the state of **d)**. You can use Dirac notation and/or the matrix notation.

**g)** Repeat question **d)** (and this implies you also need to do some work as in **c)** ), but now for the expectation value of the total angular momentum $\langle \hat{L} \rangle$. You can use Dirac notation and/or the matrix notation.

**Problem C8-9.3**
A certain atom is in a state with its total orbital angular momentum vector $\boldsymbol{L}$ (described by the operator $\hat{L}$) defined by orbital quantum number $l = 1$.

**a)** What is in this case the length of this vector for total angular momentum $\boldsymbol{L}$ ?

For the system in this state, the operator for the $z$-component of angular momentum is $\hat{L}_z$. It has three eigenvalues, $+\hbar$ (with corresponding eigenstate $|+_z\rangle$), $0\hbar$ (with eigenstate $|0_z\rangle$), and $-\hbar$ (with eigenstate $|-_z\rangle$). This operator can be represented as a matrix, and the ket-states as column vectors, using the basis spanned by $|+_z\rangle$, $|0_z\rangle$ and $|-_z\rangle$, according to

$$\hat{L}_z \leftrightarrow \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad |+_z\rangle \leftrightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |0_z\rangle \leftrightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \text{ and } \quad |-_z\rangle \leftrightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Using this same basis for the representation, the operator and eigenstates for the system's $x$-component of angular momentum are given by

$$\hat{L}_x \leftrightarrow \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad |+_x\rangle \leftrightarrow \begin{pmatrix} \frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}, \quad |0_x\rangle \leftrightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}, \text{ and } \quad |-_x\rangle \leftrightarrow \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}.$$

**b)** *Calculate* with this information what the eigenvalues are that belong to $|+_x\rangle$, $|0_x\rangle$ and $|-_x\rangle$.

**c)** At some point the system is in the normalized state $|\Psi_1\rangle = \sqrt{\tfrac{1}{8}}\,|+_z\rangle + \sqrt{\tfrac{3}{8}}\,|0_z\rangle + \sqrt{\tfrac{4}{8}}\,|-_z\rangle$. Calculate for this state the expectation value for angular momentum in $z$-direction and the expectation value for angular momentum in $x$-direction.

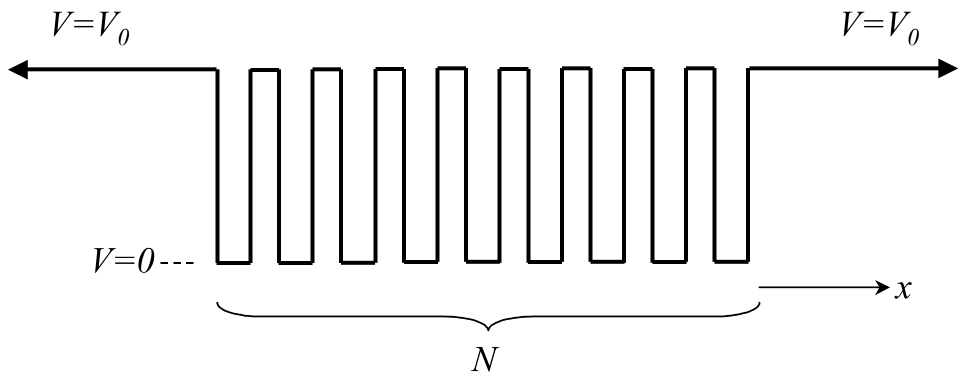
**d)** At some point the system is in the normalized state $|\Psi_2\rangle = \sqrt{\tfrac{1}{3}}\,|+_z\rangle + \sqrt{\tfrac{1}{3}}\,|0_z\rangle + \sqrt{\tfrac{1}{3}}\,|-_z\rangle$, and you are going to measure the $x$-component of the system's angular momentum. What are the possible measurement results? Calculate for each possible measurement result the probability given that the system is in state $|\Psi_2\rangle$.

**e)** At some point the system is in the state $|\Psi_3\rangle = i\,|+_z\rangle + 2\,|0_z\rangle - i\,|-_z\rangle$. Note that this state is not normalized. Calculate for this state $\langle \hat{L}_z \rangle$.

**f)** At some point the system is in the normalized state $|\Psi_4\rangle = \sqrt{\tfrac{1}{2}}\,|+_z\rangle + \sqrt{\tfrac{1}{2}}\,|-_z\rangle$. Calculate for this state the quantum uncertainty $\Delta L_z$ in the $z$-component of the system's angular momentum.

**Problem C8-9.4**

This problem is meant to clarify the link between the *particle-in-a-box* system, and the description of electrons in solid state material as treated in a course on solid state physics. Consider a one-dimensional array of $N$ potential wells, formed by the potential $V(x)$ as in the following figure. The width of the wells is $a$, the width of the barriers between the wells is $b$.




**a)** Consider the case $N = 1$, with the width of that well $a = 0.2$ nm, and $V_0 = 3$ eV. This well contains a single particle with mass $m$. Calculate for which values of $m$ this system has 2 bound energy eigenstates.

**b) READ QUESTION c) FIRST**. Consider the case $N = 2$. Assume that the system contains a particle with the same mass as considered for question **a)**. For $a = 0.2$ nm, $b = 0.1$ nm and $V_0 = 3$ eV, the tunnel coupling $T_0$ between the ground states of the left and the right well is $T_0 = 0.05$ eV. (Note that we mean here the ground states for the case that each well is not yet coupled to another well.) Make a rough sketch of the spectrum (as a function of $b$) of the bound energy eigenstates for a particle in this double well system, for the range $b = 0$ nm to $b = 10$ nm.

**c)** Add to the sketch of the spectrum of question **b)**, several of the unbound energy eigenstates. Sketch only the ones with the lowest energy eigenvalues. Put labels in the sketch to point out which energy eigenvalues are bound states, and which energy eigenvalues are unbound states.

**d)** What is the level spacing between the unbound energy eigenvalues of question **c)**?

**e)** Now consider the case where $N$ is a very large number. Repeat question **b)** for this case.

**f)** Give an example of a real physical situation where the model system considered in **e)** is relevant. Explain your answer.