Problem set with Chapter 4 for werkcollege Kwantumfysica 1

For werkcolleges in December 2010, see the schedule with detailed planning per group

Work out the problem below and these problems in the book:
4.6 (use p. 857), 4.9, 4.10 (only for $\langle p_x \rangle$), 4.15, 4.17, 4.21, 4.22, 4.33, 4.35, 4.36

(Homework was problems: 4.1, 4.2, 4.3, 4.4, 4.5, 4.11, 4.12, 4.13, 4.14, 4.16, 4.25, 4.29)

Problem C4.1
This problem continues on the derivation of the states for a particle in a box in the book, and uses notation as on p. 92.

a) What are the energy eigenfunctions for n=1 and n=2?

b) Calculate $\langle p_x \rangle$ and Δp_x for n=1 and n=2.

c) The value of the energy of the ground state E_1 is higher than the lowest potential (the bottom) of the box: $E_1>0$. Is it possible to reduce this ground state energy by reducing the uncertainty Δx? Explain qualitatively why it is not possible to reduce this ground state energy by reducing the uncertainty Δx. Hint: use the Heisenberg uncertainty relation, and use the answer for Δp_x in **b)** and a quick rough estimate for Δx and assume that the width a cannot be changed.

d) The same question as c**)**, but now for Δp_x. Explain qualitatively that this does not work out either.

e) Calculate for the ground state the expectation value for the particle's kinetic energy.

f) By definition, an energy eigenvalue of a time-independent Hamiltonian has a well-defined energy, with zero uncertainty: ΔH=0. The expression for E_1 with the momentum expressed in terms of $k_1=p_x/\hbar$ suggests that also k_1 must have a well-defined value (with zero uncertainty) for the energy eigenstate $|\varphi_1\rangle$. However, the answer on **b)** gives $\Delta p_x > 0$ and seems to contradict this. Explain why there is no contradiction, and why $\langle p_x \rangle = 0$.

P.S. Some useful integrals solved:

$$\int \mathbf{sin}(Ax)\mathbf{cos}(Ax)dx = \frac{-1}{2A}\left(\mathbf{cos}^2(Ax) - 1\right)$$

$$\int \mathbf{sin}^2(Ax)dx = \frac{1}{2A}\left(Ax - \mathbf{sin}(Ax)\mathbf{cos}(Ax)\right)$$