Problem set for Chapter 1-2 for werkcollege Kwantumfysica 1

For werkcollege of 1st week and 1st werkcollege of 2nd week (Nov. 2010)

Problems (Homework, to be made before the werkcollege)
Chapter 1 - 1.1, 1.4, 1.9, 1.10, 1.20, Chapter 2 - 2.17, 2.18

Problems to work on during werkcollege
Problems C1-2.1 – C1-2.5 (this hand out), Chapter 2 - 2.21, 2.23, 2.33

Problem C1-2.1 (Note: For this problem, assume that the constants A_i are real valued.)

The wavefunctions of particles 1-5 are (in one dimension) given by

$$\Psi_1(x,t) = A_1 e^{-x^2/4} \quad \text{(see also book p. 56)},$$

$$\Psi_2(x,t) = A_2 e^{-x^2/b} e^{3i} \ ,$$

$$\Psi_3(x,t) = A_3 e^{-|2x|} \ ,$$

$$\begin{cases} \Psi_4(x,t) = A_4 e^{i\omega t}, & |x| \le a \\ \Psi_4(x,t) = 0, & |x| > a \end{cases} ,$$

$$\begin{cases} \Psi_5(x,t) = A_5 \cos\left(\pi \dfrac{x}{2a}\right) e^{i\omega t}, & |x| \le a \\ \Psi_5(x,t) = 0, & |x| > a \end{cases} .$$

a) For all 5 cases, write out the normalized wavefunctions and sketch the probability densities versus x.

b) For Ψ_4, assume a=2, and calculate the probability that measurement of x gives a result between 1 and 2.

c) For Ψ_5, assume a=2, and calculate the probability that measurement of x gives a result between 1 and 2.

d) Which physical situation gives a wavefunction in the form of Ψ_5?

Problem C1-2.2
A single electron is in a large empty space, where no fields or external forces act on this "free particle". We will only look at its behavior along the x-direction. At some time $t = t_o$, the wavefunction that describes the linear momentum p_x of the electron is

$$\Psi_p(p_x,t) = Ce^{-(p_x/a)^2} \ .$$

a) If we would now measure the electron's velocity, what value are we most likely going to find?

b) We decided not to measure the velocity, because we did not know well enough yet where the electron was. What was at t_o approximately the lower limit in the fundamental uncertainty in the x-position of the electron, expressed in meters? Use $a = 9.109 \cdot 10^{-31}$ kg m s^{-1}.

c) We are happy with the answer on **b)**, the electron is well enough localized in the volume where we can do the experiments. However, our thinking about it took 100 seconds. Estimate how much the lower limit in the uncertainty in the x-position of the electron increased during this time.

d) We decide to measure now as soon as possible. What is now the value for the electron's velocity that we are most likely going to find?

Problem C1-2.3
A superconducting loop can have so little damping that the electrical current in the loop behaves quantum mechanically. We consider her a particular system that can only be in two states: a state in which the current I flows clockwise (+1 μA), or a state in which the current I flows counter clockwise (-1 μA). For both of these states the amplitude of the current is 1 μA. The quantum state of the loop can be a superposition of these two states. The wavefunction for the system in the clockwise state is $\Psi(I,t) = \varphi_R$, for the other state it is $\Psi(I,t) = \varphi_L$.

a) The system is prepared in a quantum state such that the probability for it to be in one of the two current states is 50%. Sketch a probability density $P(I) = |\Psi(I,t)|^2$. Write down a normalized wavefunction in terms of φ_R and φ_L that is in agreement with this situation (more answers possible).

b) The apparatus for preparing this state maybe needs to be calibrated. It maybe has an offset (that is stable in time), which causes that the prepared superposition state corresponds to probabilities (50+A)% for the φ_R state and (50-A)% for the φ_L state. To check this, we will prepare the system many times in the same way and measure it. This can be used to see what the value for A is, and one can thus check whether A is very close to zero. The current in the loop is measured by turning on the coupling to a device that can measure the current in the loop. This is done after the preparation step. What are the possible current values on the display of the measuring device?

c) The experimentalists find out that the offset is large, and find the counter-clockwise state 25% of the time. Write down a normalized wavefunction in terms of φ_R and φ_L that is in agreement with this situation (more answers possible).

Problem C1-2.4
We will perform a double slit experiment with electrons as in Fig. 2.16 in the book. The width of the slits is a, and they have very sharp edges. The distance between the slits (center to center) is d. The incoming electron flux (a very wide bundle much wider than d) is hitting the two slits at an equal rate. You can also assume that the plate that contains the slits is very thin (less than a, but thick enough to block electrons).

a) Sketch a probability density for the wavefunction that describes the y-position of the electrons, for the moment they just come out of the slits (assume that the plane with the slits is very thin, that is, the electrons spend almost zero time between the walls of the slits). Write down a normalized wavefunction that is in agreement with it.

b) The electrons that fly through the slits have been accelerated (starting with zero velocity) over 10 kV, in a region with an electric field. What is the De Broglie wavelength of the electrons?

c) We will now assume that $d >> a$, and that a and d are both much smaller than the distance to the screen on the right. Further, we put an electron detector with a very narrow opening on the plane on the right, at the symmetry axis at $y = 0$. Inside one of the slits (the higher one in the figure, named slit A, the other one is slit B), we will build a device that can be used for changing the phase of the complex wavefunction that describes the x-position of the electrons. If an electron passes through the slit A, the phase of its wavefunction $\Psi(x,t)$ increases by a controllable amount φ, according to

$$\Psi(x,t) \rightarrow \Psi(x,t)e^{i\varphi}$$

With a constant electron flux going towards the slits, we count the number electron clicks per second from the detector. Analyze and sketch how this count rate varies as a function of φ, with slit B closed (only A open).

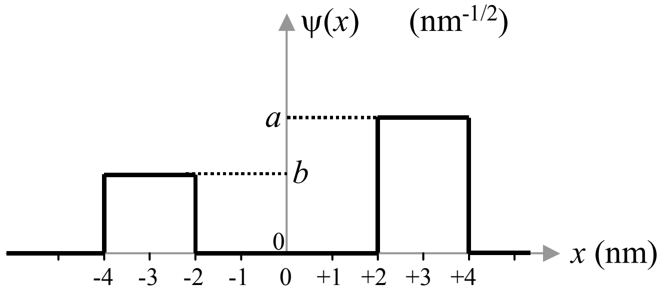
d) The same, but now with both slits open.

e) How rapidly the phase of electron wavefunctions evolve in time depends on the material that surrounds the electron. Explain using the answers on the previous questions how one can build an electron microscope with a double-slit setup.

Problem C1-2.5

Note: This problem uses the concept *expectation value*, which was treated this week in the "hoorcollege", and introduced in the book at page 76 of Chapter 3. So look it up first if needed.

The position x of a particle is described by the normalized, real-valued wavefunction as sketched in the figure below, with $a = \sqrt{3/10\ \text{nm}^{-1}}$ and $b = \sqrt{2/10\ \text{nm}^{-1}}$.



a) What is the expectation value $\langle \hat{x} \rangle$ for this state?

b) Show that the uncertainty Δx in the particle's position is $\sqrt{\frac{5384}{600}}$ nm.

c) With the particle's wavefunction as sketched, you plan to measure the position x. What is the probability for detecting a value in the range $\langle \hat{x} \rangle$ - 0.1 nm $< x < \langle \hat{x} \rangle$ + 0.1 nm?

d) With the particle's wavefunction as sketched, you plan to measure the position x. What is the probability for detecting a value in the range -4 nm $< x <$ -3 nm?