Kwantumfysica 1

Overview of concepts that should be understood

Version of 30 January 2008 (this new version includes concepts of H9.1 - H9.3)
Caspar van der Wal – c.h.van.der.wal@rug.nl

For the exam you should be familiar with the following concepts. You should be able to answer theory questions about them, and to apply them in problems about physical systems (order of presentation here is not related to order in the book).

Postulates of quantum mechanics
Formulating a theory for physics on the basis of postulates
Position of quantum theory with respect to classical theories, relativistic theories, thermo-dynamical theory

Wavefunction describes the state of a system, contains *all* information
Wavefunction interpretation, relation to probability density
Superposition principle, superposition of states of a system
Physical properties are describes by an operator, an observable
Time-dependent Schrödinger equation
Measuring a quantum system
Probabilities for measurement outcomes
Ensemble average versus expectation value for a single system

Describing physical systems
Number of degrees of freedom of a system
Generalized coordinates
Conjugate variables
Quantizing a system (defining the relevant operators for a system, from determining the number of degrees of freedom, and the classical variables that describe the dynamics)

Representation of states and notation
Wavefunction notation and Dirac notation
x-representation, p_x-representation, representation in terms of eigenstates of an arbitrary operator (for example energy eigenstates)
Fourier transform relation between x-representation, p_x-representation
Dirac delta function and its properties in integrals
"Spectral decomposition" of an arbitrary state into eigenstates of an operator
Transforming wavefunction notation into Dirac notation and vice versa
Inner product in Dirac notation and wavefunction notation, and the meaning of it
Expectation value for an operator in Dirac notation
Fourier examples with Gaussian-, block- and sinc-function states.
Basis
Hilbert space
Complete Set of Commuting Observables to get vectors that span the Hilbert space

Operators
How to calculate physical properties of a system in a certain state with operators
Eigenvalue equation
Eigenvalues
Eigenstates
Eigenstates of an observable are orthogonal, eigenvalues are real
Hermitian operators
Hermitian adjoint of an operator, properties and algebraic relations for this
Operators which are functions (and Taylor series) of another operator
Examples of operators
- Position operator
- Momentum operator
- Operator for total energy (Hamiltonian)

Commutator bracket
Commutator algebra
Meaning of the fact that two operators commute / do not commute
Relation with Heisenberg uncertainty relation
Time evolution described using $\frac{d\langle \hat{A} \rangle}{dt} = \frac{i}{\hbar}\langle \Psi | [\hat{H}, \hat{A}] | \Psi \rangle$
Ehrenfest principle
Constants of motion – operator commutes with Hamiltonian
Conservation laws

Parity, and parity operator

Time-evolution
Hamiltonian
Bound states
Discrete versus continuous systems
Systems with a time-independent (stationary) Hamiltonian
Energy eigenvalues and eigenvectors
Deriving time-independent Schrödinger eq. from time-dependent Schrödinger eq.
Stationary states (and meaning for closed system versus open system)
Expectation value as a function of time for an arbitrary observable $\hat{A}$.
Oscillations of a system
Deriving photon energy is $\hbar\omega_{nm} = E_n - E_m$, radiative emission from a system
Describing time-evolution with operator $\hat{U} = e^{\frac{-i\hat{H}t}{\hbar}}$ for kets and $\hat{U}^+ = e^{\frac{+i\hat{H}t}{\hbar}}$ bra's
Time evolution of wavepackets

Wavefunction of a particle moving in 1 dimension
Free particle versus bound particle
Expectation value $\langle x \rangle$ for a state $\Psi(x)$
Uncertainty Δx for a state $\Psi(x)$
Normalizing a state $\Psi(x)$
Heisenberg uncertainty relation
Probability for measurement results if you measure x on a system in the state $\Psi(x)$
State of a system after a measurement
Wavepackets

Double-slit experiments
Quantum interference of the wavefunction of a system
Wave-particle duality for photons, big macroscopic hard balls, electrons

Wave mechanics
Broglie wavelength
Wave packets
Phase velocity and group velocity
Wave mechanics for plane waves incident on a tunnel barrier, step-potential
Tunnel effect

Particle in a box
Its energy eigenstates and eigenvalues
Particle in a infinite / finite potential well
2D box
2 particles in a single 1D box
Coupling two quantum wells with in total one particle in the system

Harmonic oscillator
Its energy eigenstates and eigenvalues
Simple 1D Harmonic oscillator
Annihilation/destruction and creation operators
Energy quanta (photons) in this system
Formulated in x-space and k-space
2D harmonic oscillator

Coupling quantum wells
Linear Combination of Atomic Orbitals
How solid-state physics (energy bands) emerges from the physics of the 1D potential well.

Systems with more than one degree of freedom
Simple examples of systems with 2 particles
Simple examples of systems with 1 particle in a 2D or 3D potential
Basis states to describe these systems

Degeneracy and identical particles
As a result of exchanging identical particles in a system with two particles
As a result of symmetry in the 2D or 3D potential of a single particle
Accidental degeneracy
Symmetric and antisymmetric states
Exchange

Quantum mechanical treatment of angular momentum
Operators for total, and x-, y-, and z-component of angular momentum
Intrinsic (spin) versus orbital angular momentum
General form of eigenvalues and eigenfunctions of angular momentum operators
Commutation and uncertainty relations between angular momentum operators
Rotation operator
Spherical harmonics

NOTE:
Fourier transform between x-representation and k-representation is not treated at a specific location in the book, but you get a good overview following the sections listed here (see also hoorcollege slides):

p. 100 and 117	Expansion into a series of discrete states
p. 102	Projection onto k-states (plane waves)
p. 125	Delta function in x-representation
p. 122 and 158	Square wave in x-representation
p. 156	Wave packet in x- and p_x-representation
p. 160	Gaussian in x- and p_x-representation
p. 211	Applied to the harmonic oscillator states
p. 849 and 857	Appendix A, C

Note on notation for Fourier transforms:
The book uses $b(k)$ to denote the Fourier transform of $\varphi(x)$. The function $\varphi(x)$ is then a superposition of (integral over) plane waves with amplitude $b(k)$. The book denotes these plane waves as φ_k. This leads to the following (somewhat confusing) notation for the pair of Fourier transform equations,

$$\varphi(x) = \int_{-\infty}^{\infty} b(k)\,\varphi_k\, dk \quad ,$$
$$b(k) = \int_{-\infty}^{\infty} \varphi(x)\,\varphi_k^*\, dx$$

where the plane waves are denoted as [see Eq. (4.40), p. 102, and Eq. (5.25), p. 122]

$$\varphi_k(x) = \varphi_k = \frac{1}{\sqrt{2\pi}} e^{ikx} \quad .$$
$$\varphi_k^*(x) = \varphi_k^* = \frac{1}{\sqrt{2\pi}} e^{-ikx}$$

It is, however, conventional to use a related symbol for a function and its Fourier transform, for example as

$$\varphi(x) \overset{Fourier}{\leftrightarrow} \overline{\varphi}(k) \ , \qquad \text{or} \qquad \Psi(x) \overset{Fourier}{\leftrightarrow} \overline{\Psi}(k).$$

One also usually keeps the plane waves expressions explicitly written in the Fourier transform integrals. This notation is used for the lecture slides, the handout on Fourier transforms from the Cohen-Tannoudji book, and the problem sets. This gives for example the following pair of equations for the Fourier relation between x-representation and k-representation of a state:

$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \overline{\Psi}(k) e^{ikx}\, dk \quad .$$
$$\overline{\Psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x) e^{-ikx}\, dx$$