

Kwantumfysica 1 Years 2007-2008 and later

Overview of concepts that should be understood

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Additional note for lecturers of related courses

The course **Kwantumfysica 1** (2nd quarter of the second bachelor year for Physics and Astronomy students) uses **Part I** and half a chapter of Part II of the book:

Introductory Quantum Mechanics
R. L. Liboff, 4th edition, Addison Wesley, ISBN 0-8053-8714-5

Below a list with concepts that the students should understand (I also give this list to the students).

In case you like to suggest some other topics that need to be treated in the course, please keep in mind that I think that the course is a bit too full already. Also, given the book that we use, we face the following limitations:

Part I of the book is completely in the *Schrödinger picture* (implicitly, it is never mentioned yet)

Angular momentum is not used at all in this Part I. Hence, also *spin* is still completely out of the picture. In the year 2007-2008, it was decided to treat angular momentum shortly at the end of the course (first half of Chapter 9), such that the course Kwantumfysica 2 can go a bit faster in the beginning.

Part I mainly presents the material using continuous systems with one degree of freedom (in the end a little bit systems with two degrees of freedom), and a stationary Hamiltonian. Discrete states only appear as energy eigenstates of a bound system.

As a consequence of the above, the book does not formulate the theory yet in terms of matrix mechanics (which is so nice for making tractable little problems where you don't have to integrate all the time). Part I hardly allows for making problems with discrete systems where one formulates operators and states in terms of matrices and vectors. I do already present this a bit at the hoorcollege though, where one can relate simple examples to the theory in the book.

Further, all states are *pure quantum states*, no mixtures (density matrix) are used yet.

I treat all of Part I of the book as *study material* (what they *must* know for the exam) or *reading material* (what I encourage them to read, and present a little bit in the hoorcollege, but what is not core material for the exam). I also hand out some copies with additional reading material.

The parts which I decided to make *reading material* are:

- Chapter 2: Detailed treatment of historical experiments (photoelectric effect, blackbody radiation, etc.)
- Chapter 2: Short note on hidden variable theories
- Chapter 2 and 7: Feynman path integrals, principle of least action
- Chapter 7: WKB method
- Chapter 7: Some of many sections on wave mechanics. Tunneling and the step-potential is treated, but I skip the Ramsauer effect and a detailed section on scattering from a barrier.
- Chapter 8: I do treat Linear Combination of Atomic Orbitals, and want them to understand two figures on how energy bands emerge from atomic levels if you couple many atoms. But several sections which are basically solid-state physics are reading material. These reading sections treat: periodic potential from a lattice, Bloch wavefunctions, quasi-momentum, energy bands, Bragg reflection, theory of electrical conduction in solids, density of states, etc.

Overview of concepts that should be understood

For the exam you should be familiar with the following concepts. You should be able to answer theory questions about them, and to apply them in problems about physical systems (order of presentation here is not related to order in the book).

Postulates of quantum mechanics

Formulating a theory for physics on the basis of postulates
Position of quantum theory with respect to classical theories, relativistic theories, thermo-dynamical theory

Wavefunction describes the state of a system, contains *all* information

Wavefunction interpretation, relation to probability density

Superposition principle, superposition of states of a system

Physical properties are describes by an operator, an observable

Time-dependent Schrödinger equation

Measuring a quantum system

Probabilities for measurement outcomes

Ensemble average versus expectation value for a single system

Describing physical systems

Number of degrees of freedom of a system

Generalized coordinates

Conjugate variables

Quantizing a system (defining the relevant operators for a system, from determining the number of degrees of freedom, and the classical variables that describe the dynamics)

Representation of states and notation

Wavefunction notation and Dirac notation

x -representation, p_x -representation, representation in terms of eigenstates of an arbitrary operator (for example energy eigenstates)

Fourier transform relation between x -representation, p_x -representation

Dirac delta function and its properties in integrals

“Spectral decomposition” of an arbitrary state into eigenstates of an operator

Transforming wavefunction notation into Dirac notation and vice versa

Inner product in Dirac notation and wavefunction notation, and the meaning of it

Expectation value for an operator in Dirac notation

Fourier examples with Gaussian-, block- and sinc-function states.

Basis

Hilbert space

Complete Set of Commuting Observables to get vectors that span the Hilbert space

Operators

How to calculate physical properties of a system in a certain state with operators

Eigenvalue equation

Eigenvalues

Eigenstates

Eigenstates of an observable are orthogonal, eigenvalues are real

Hermitian operators

Hermitian adjoint of an operator, properties and algebraic relations for this

Operators which are functions (and Taylor series) of another operator

Examples of operators

Position operator

Momentum operator

Operator for total energy (Hamiltonian)

Commutator bracket

Commutator algebra

Meaning of the fact that two operators commute / do not commute

Relation with Heisenberg uncertainty relation

Time evolution described using $\frac{d\langle\hat{A}\rangle}{dt} = \frac{i}{\hbar}\langle\Psi|[\hat{H}, \hat{A}]\Psi\rangle$

Ehrenfest principle

Constants of motion – operator commutes with Hamiltonian

Conservation laws

Parity, and parity operator

Time-evolution

Hamiltonian

Bound states

Discrete versus continuous systems

Systems with a time-independent (stationary) Hamiltonian

Energy eigenvalues and eigenvectors

Deriving time-independent Schrödinger eq. from time-dependent Schrödinger eq.

Stationary states (and meaning for closed system versus open system)

Expectation value as a function of time for an arbitrary observable \hat{A} .

Oscillations of a system

Deriving photon energy is $\hbar\omega_{nm} = E_n - E_m$, radiative emission from a system

Describing time-evolution with operator $\hat{U} = e^{\frac{-i\hat{H}t}{\hbar}}$ for kets and $\hat{U}^\dagger = e^{\frac{+i\hat{H}t}{\hbar}}$ bra's

Time evolution of wavepackets

Wavefunction of a particle moving in 1 dimension

Free particle versus bound particle

Expectation value $\langle x \rangle$ for a state $\Psi(x)$

Uncertainty Δx for a state $\Psi(x)$

Normalizing a state $\Psi(x)$

Heisenberg uncertainty relation

Probability for measurement results if you measure x on a system in the state $\Psi(x)$

State of a system after a measurement

Wavepackets

Double-slit experiments

Quantum interference of the wavefunction of a system

Wave-particle duality for photons, big macroscopic hard balls, electrons

Wave mechanics

Broglie wavelength

Wave packets

Phase velocity and group velocity

Wave mechanics for plane waves incident on a tunnel barrier, step-potential

Tunnel effect

Particle in a box

Its energy eigenstates and eigenvalues

Particle in a infinite / finite potential well

2D box

2 particles in a single 1D box

Coupling two quantum wells with in total one particle in the system

Harmonic oscillator

Its energy eigenstates and eigenvalues

Simple 1D Harmonic oscillator

Annihilation/destruction and creation operators

Energy quanta (photons) in this system

Formulated in x -space and k -space

2D harmonic oscillator

Coupling quantum wells

Linear Combination of Atomic Orbitals

How solid-state physics (energy bands) emerges from the physics of the 1D potential well.

Systems with more than one degree of freedom

Simple examples of systems with 2 particles

Simple examples of systems with 1 particle in a 2D or 3D potential

Basis states to describe these systems

Degeneracy and identical particles

As a result of exchanging identical particles in a system with two particles

As a result of symmetry in the 2D or 3D potential of a single particle

Accidental degeneracy

Symmetric and antisymmetric states

Exchange

Quantum mechanical treatment of angular momentum

Operators for total, and x -, y -, and z -component of angular momentum

Intrinsic (spin) versus orbital angular momentum

General form of eigenvalues and eigenfunctions of angular momentum operators

Commutation and uncertainty relations between angular momentum operators

Rotation operator

Spherical harmonics

NOTE:

Fourier transform between x -representation and k -representation is not treated at a specific location in the book, but you get a good overview following the sections listed here (see also hoorcollege slides):

- p. 100 and 117 Expansion into a series of discrete states
- p. 102 Projection onto k -states (plane waves)
- p. 125 Delta function in x -representation
- p. 122 and 158 Square wave in x -representation
- p. 156 Wave packet in x - and p_x -representation
- p. 160 Gaussian in x - and p_x -representation
- p. 211 Applied to the harmonic oscillator states
- p. 849 and 857 Appendix A, C

Note on notation for Fourier transforms:

The book uses $b(k)$ to denote the Fourier transform of $\varphi(x)$. The function $\varphi(x)$ is then a superposition of (integral over) plane waves with amplitude $b(k)$. The book denotes these plane waves as φ_k . This leads to the following (somewhat confusing) notation for the pair of Fourier transform equations,

$$\varphi(x) = \int_{-\infty}^{\infty} b(k) \varphi_k dk$$
$$b(k) = \int_{-\infty}^{\infty} \varphi(x) \varphi_k^* dx$$

where the plane waves are denoted as [see Eq. (4.40), p. 102, and Eq. (5.25), p. 122]

$$\varphi_k(x) = \varphi_k = \frac{1}{\sqrt{2\pi}} e^{ikx}$$
$$\varphi_k^*(x) = \varphi_k^* = \frac{1}{\sqrt{2\pi}} e^{-ikx}$$

It is, however, conventional to use a related symbol for a function and its Fourier transform, for example as

$$\varphi(x) \overset{\text{Fourier}}{\leftrightarrow} \bar{\varphi}(k) \quad , \quad \text{or} \quad \Psi(x) \overset{\text{Fourier}}{\leftrightarrow} \bar{\Psi}(k).$$

One also usually keeps the plane waves expressions explicitly written in the Fourier transform integrals. This notation is used for the lecture slides, the handout on Fourier transforms from the Cohen-Tannoudji book, and the problem sets. This gives for example the following pair of equations for the Fourier relation between x -representation and k -representation of a state:

$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{\Psi}(k) e^{ikx} dk$$
$$\bar{\Psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x) e^{-ikx} dx$$