Kwantumfysica 1 - Handout on Fourier transforms

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The Fourier transform between x-representation and k-representation is not treated at a specific location in the book, but you get a good overview by going through the following the sections (see also hoorcollege slides):

p. 100 and 117	Expansion into a series of discrete states
p. 102	Projection onto k-states (plane waves)
p. 125	Delta function in x-representation
p. 122 and 158	Square wave in x-representation (also defining the *sinc* function)
p. 156	Wave packet in x- and p_x-representation
p. 160	Gaussian in x- and p_x-representation
p. 211	Applied to the harmonic oscillator states
p. 849 and 857	Appendix A, C

Note on notation for Fourier transforms:

The book uses $b(k)$ to denote the Fourier transform of $\varphi(x)$. The function $\varphi(x)$ is then a superposition of (integral over) plane waves with amplitude $b(k)$. The book denotes these plane waves as φ_k. This leads to the following (somewhat confusing) notation for the pair of Fourier transform equations,

$$\varphi(x) = \int_{-\infty}^{\infty} b(k)\,\varphi_k\,dk \quad ,$$

$$b(k) = \int_{-\infty}^{\infty} \varphi(x)\,\varphi_k^*\,dx$$

where the plane waves are denoted as [see Eq. (4.40), p. 102, and Eq. (5.25), p. 122]

$$\varphi_k(x) = \varphi_k = \frac{1}{\sqrt{2\pi}} e^{ikx} \quad .$$

$$\varphi_k^*(x) = \varphi_k^* = \frac{1}{\sqrt{2\pi}} e^{-ikx}$$

It is, however, conventional to use a related symbol for a function and its Fourier transform, for example as

$$\varphi(x) \overset{Fourier}{\leftrightarrow} \overline{\varphi}(k) \;, \quad \text{or} \quad \Psi(x) \overset{Fourier}{\leftrightarrow} \overline{\Psi}(k), \quad \text{or} \quad x(t) \overset{Fourier}{\leftrightarrow} X(\omega)\,.$$

One also usually keeps the plane waves expressions explicitly written in the Fourier transform integrals. This notation is used for the lecture slides, the handout on Fourier transforms from the Cohen-Tannoudji book, and the problem sets. This gives for example the following pair of equations for the Fourier relation between x-representation and k-representation of a state:

$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \overline{\Psi}(k) e^{ikx}\,dk \quad .$$

$$\overline{\Psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x) e^{-ikx}\,dx$$

In some engineering books, they use the convention (which also works if you use it consistently) to use the prefactors 1 and 1/(2π) instead of 1/(√2π) and 1/(√2π).